American Book Company's

PASSING THE TEXAS ALGEBRA I END-OF-COURSE ASSESSMENT

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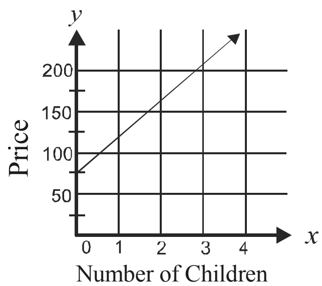
Diagnostic Test

1. The graph shows the price of an entry ticket into *The Big Adventure Theme Park* for one day with one adult and x number of children.

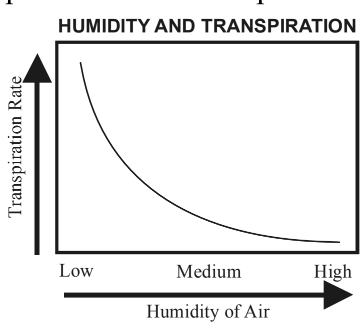


Which equation best represents the graph?

A $y = 75x + 45$
B $y = 45x + 75$
C $y = 75x$
D $y = 45x + 45$
E $y = 45x$

A.1.D

2. Transpiration is the process by which a plant loses water vapor through its leaves.



According to the graph, which of the following would be an accurate conclusion?

F On days of low humidity, the transpiration rate is highest.
G On days of medium humidity, the transpiration rate is above average.
H On very humid days, the transpiration rate is highest.
J On days of low humidity, the transpiration rate is lowest.
K It is very humid everyday.

A.1.E

3. A scientist wants to determine the half life of iodine-131 experimentally. He started with 1 gram of iodine-131 and recorded the day when half of the iodine decomposed. He recorded the following data which shows that the mass decayed by half every eight days.

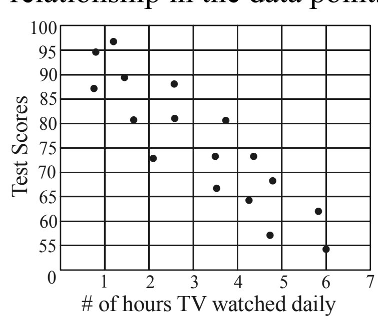
Half-life (n)	Mass
0	1 gram
1	0.5 gram
2	0.25 gram
3	0.125 gram

Which of the following functions describes the decay of mass for iodine-131?

A $f(n) = 2^n$
B $f(n) = \frac{1}{2}n$
C $f(n) = \dfrac{1}{n}$
D $f(n) = \dfrac{1}{2^n}$
E $f(n) = \dfrac{2}{n}$

A.1.D

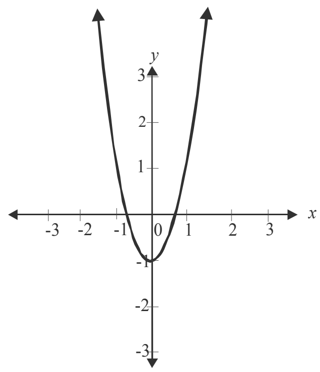
4. Which of the following best describes the relationship in the data points shown below?



F Positive relationship
G Negative relationship
H No relationship
J Quadratic relationship
K Cannot be determined

A.1.B

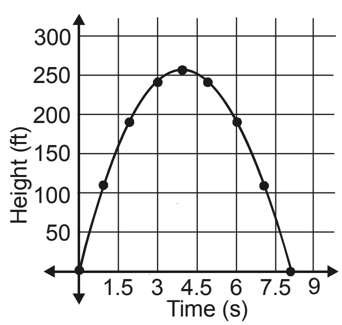
43. The graph below is a quadratic function in the form of $y = ax^2 + c$. What are the values of a and c?



A $a = -1, c = -1$
B $a = 1, c = 1$
C $a = 2, c = -1$
D $a = 2, c = 1$
E $a = 1.5, c = 1.5$

A.9.D

44. An arrow is shot upward with an initial velocity of 128 feet per second. The height (h) of the arrow is a function of time (t) in seconds since the arrow left the ground and can be expressed by the equation $h = 128t - 16t^2$.



What is the height of the arrow after 7.5 seconds?

F $-13,440$ ft
G 840 ft
H 0 ft
J 8 ft
K 60 ft

A.10.A

45. Solve for x: $x^2 - 3x - 10 = 0$

A $x = 5, 2$
B $x = 5, -2$
C $x = -5, 2$
D $x = -5, -2$
E No real solution

A.10.A

46. Changing the quadratic equation from $y = 2x^2$ to $y = -\frac{1}{2}x^2$ will:

F invert the parabola
G invert the parabola, flatten and widen the curve
H makes the parabola taller and thinner
J moves the parabola down $\frac{1}{2}$ unit
K make the parabola wider and flatter

A.9.B

47. The graph of $y = x^2 + 2$ is changed to $y = x^2 - 2$. What happened to the graph?

A It moved up four spaces along the y-axis.
B It moved down two spaces along the y-axis.
C It moved down four spaces along the y-axis.
D It became wider.
E It moved to the left 4 spaces along the x-axis.

A.9.C

48. It takes 40 minutes for 3 copiers to finish a printing job. How long would it take for the same job if 4 copiers were working on the printing job? All the copiers are identical and print at the same speed.

A.11.B

Evaluation Chart for the Diagnostic Mathematics Test

Directions: On the following chart, circle the question numbers that you answered incorrectly. Then turn to the appropriate topics (listed by chapters), read the explanations, and complete the exercises. Review the other chapters as needed. Finally, complete the *Passing the Texas Algebra I End-of-Course Assessment* Practice Tests to further review.

		Questions	Pages
Chapter 1:	Exponents		13–17
Chapter 2:	Introduction to Algebra	28	18–27
Chapter 3:	Solving Multi-Step Equations and Inequalities	20, 30, 33, 36	28–37
Chapter 4:	Algebra Word Problems	31, 34	38–48
Chapter 5:	Polynomials	26	49–72
Chapter 6:	Factoring		73–82
Chapter 7:	Solving Quadratic Equations	45	83–94
Chapter 8:	Graphing and Writing Equations and Inequalities	9, 11, 12, 17, 18	95–115
Chapter 9:	Applications of Graphs	1, ,2, 4, 5, 6, 13, 14, 15, 22, 41, 43, 46, 47, 49, 50	116–142
Chapter 10:	Systems of Equations	10, 16, 32, 35, 37, 38, 39, 40	143–156
Chapter 11:	Relations and Functions	7, 8, 21, 23, 24, 27, 29, 44	157–184
Chapter 12:	Proportions and Patterns	3, 19, 25, 42, 48	185–200

Chapter 2
Introduction to Algebra

This chapter covers the following TX Algebra I standards:

Objective 1:	Foundation for Functions	A.1.C A.1.D
Objective 2:	Foundation for Functions	A.3.A

2.1 Algebra Vocabulary

Vocabulary Word	Example	Definition
variable	$4x$ (x is the variable)	a letter that can be replaced by a number
coefficient	$4x$ (4 is the coefficient)	a number multiplied by a variable or variables
term	$5x^2 + x - 2$ ($5x^2$, x, and -2 are terms)	numbers or variables separated by $+$ or $-$ signs
constant	$5x + 2y + 4$ (4 is a constant)	a term that does not have a variable
degree	$4x^2 + 3x - 2$ (the degree is 2)	the largest power of a variable in an expression
leading coefficient	$4x^2 + 3x - 2$ (4 is the leading coefficient)	the number multiplied by the term with the highest power
sentence	$2x = 7$ or $5 \leq x$	two algebraic expressions connected by $=$, $\neq$, $<$, $>$, $\leq$, $\geq$, or $\approx$
equation	$4x = 8$	a sentence with an equal sign
inequality	$7x < 30$ or $x \neq 6$	a sentence with one of the following signs: $\neq$, $<$, $>$, $\leq$, $\geq$, or $\approx$
base	6^3 (6 is the base)	the number used as a factor
exponent	6^3 (3 is the exponent)	the number of times the base is multiplied by itself

2.4 Setting Up Algebra Word Problems

So far, you have seen only the first part of algebra word problems. To complete an algebra problem, an equal sign must be added. The words "**is**" or "**are**" as well as "**equal(s)**" signal that you should add an equal sign.

Example 4: Double Jake's age, x, minus 4 is 22.

$$2x \quad - \quad 4 \quad = \quad 22$$

Translate the following word problems into algebra equations. DO NOT find the solutions to the problems yet.

1. Triple the original number, n, is $2,700$.
2. The product of a number, y, and 5 is equal to 15.
3. Four times the difference of a number, x, and 2 is 20.
4. The total, t, divided into 5 groups is 45.
5. The number of parts in inventory, p, minus 54 parts sold today is 320.
6. One-half an amount, x, added to \$50 is \$262
7. One hundred seeds divided by 5 rows equals n number of seeds per row.
8. A number, y, less than 50 is 82.
9. His base pay of \$200 increased by his commission, x, is \$500.
10. Seventeen more than half a number, h, is 35.
11. This month's sales of \$$2,300$ are double January's sales, x.
12. The quotient of a number, w, and 4 is 32.
13. Six less a number, d, is 12.
14. Four times the sum of a number, y, and 10 is 48.
15. We started with x number of students. When 5 moved away, we had 42 left.
16. A number, b, divided into 36 is 12.

3.5 Multi-Step Algebra Problems

You can now use what you know about removing parentheses, combining like terms, and solving simple algebra problems to solve problems that involve three or more steps. Study the examples below to see how easy it is to solve multi-step problems.

Example 6: $3(x+6) = 5x - 2$

Step 1: Use the distributive property to remove parentheses. $3x + 18 = 5x - 2$

Step 2: Subtract $5x$ from each side to move the terms with variables to the left side of the equation.

$$\begin{array}{rl} 3x + 18 & = 5x - 2 \\ -5x \qquad & \quad -5x \\ \hline -2x + 18 & = -2 \end{array}$$

Step 3: Subtract 18 from each side to move the integers to the right side of the equation.

$$\begin{array}{rl} -18 & \quad -18 \\ \hline \end{array}$$

Step 4: Divide both sides by -2 to solve for x.

$$\frac{-2x}{-2} = \frac{-20}{-2}$$

$$x = 10$$

Example 7: $\dfrac{3(x-3)}{2} = 9$

Step 1: Use the distributive property to remove parentheses. $\dfrac{3x-9}{2} = 9$

Step 2: Multiply both sides by 2 to eliminate the fraction. $\dfrac{\not{2}(3x-9)}{\not{2}} = 2(9)$

Step 3: Add 9 to both sides, and combine like terms.

$$\begin{array}{rl} 3x - 9 & = 18 \\ +9 & \quad +9 \\ \hline \end{array}$$

Step 4: Divide both sides by 3 to solve for x.

$$\frac{3x}{3} = \frac{27}{3}$$

$$x = 9$$

Solve the following multi-step algebra problems.

1. $2(y-3) = 4y + 6$

2. $\dfrac{2(a+4)}{2} = 12$

3. $\dfrac{10(x-2)}{5} = 14$

4. $\dfrac{12y - 18}{6} = 4y + 3$

5. $2x + 3x = 30 - x$

6. $\dfrac{2a+1}{3} = a + 5$

7. $5(b-4) = 10b + 5$

8. $-8(y+4) = 10y + 4$

4.1 Real-World Linear Equations

Linear equations are very useful mathematical tools. They allow us to show relationships between two variables.

Example 2: A local cell phone company uses the equation $y = \frac{5}{2}x + 10$ to determine the charges for usage where $y =$ the cost and $x =$ the minutes used. How much will Jessica's bill be if she talked for 40 minutes?

Step 1: Substitute the known value in for x.
$y = \frac{5}{2}(40) + 10$

Step 2: Simplify.
$y = 100 + 10 = 110$
Jessica's bill will be $\$110$.

Example 3: Vincent bought a luxury car for $\$165,000$ and its value has depreciated linearly. After 5 years the value was $\$137,000$. What is the amount of yearly depreciation?

Step 1: First find how much the car's value depreciated in 5 years.
$\$165,000 - \$137,000 = \$28,000$

Step 2: Next, find the yearly depreciation by dividing $\$28,000$ by the amount of years, 5.
$\$28,000 \div 5 = \$5,600$
The value of Vincent's car depreciated $\$5,600$ each year.

Example 4: In 1990, the average cost of a new house was $\$123,000$. By the year 2000, the average cost of a new house was $\$134,150$. Based on a linear model, what is the predicted average cost for 2008?

Step 1: First, we need to find the difference between the average cost of a new house in the year 1990 and the average cost of a new house in the year 2000.
$\$134,150 - \$123,000 = \$11,150$

Step 2: Next, we need to find how much the average cost of a new house went up each year. Since it had been 10 years, divide the difference between the value in 2000 and 1990 by 10.
$\$11,150 \div 10 = \$1,115$

Step 3: Multiply the amount the average cost of a new house went up each year by the number of years between 2000 and 2008.
$\$1,115 \times 8 = \$8,920$

Step 4: Lastly, add the average cost of a new house in the year 2000 with the amount found in step 3.
$\$134,150 + \$8,920 = 143,070$
$\$143,070$ is the predicted average cost of a new house for 2008.

5.9 Monomial Roots with Remainders

Monomial roots which are not easily simplified under the square root symbol will also sometimes be encountered. Powers may be raised to odd numbers. In addition, the coefficients may not be perfect squares. Follow the example below to understand how to simplify these types of problems.

Example 11: Simplify $\sqrt{40x^7y^{11}z^{23}}$

Step 1: Begin by simplifying the coefficient. $\sqrt{40} = (\sqrt{4})(\sqrt{10})$, $\sqrt{4} = 2$, so $\sqrt{40} = 2\sqrt{10}$

Step 2: Simplify the variable with exponents.

$$\sqrt{x^7} = \left(\sqrt{x^6}\right)(\sqrt{x}),\ \sqrt{x^6} = x^3, \text{ so } \sqrt{x^7} = x^3\sqrt{x}$$

$$\sqrt{y^{11}} = \left(\sqrt{y^{10}}\right)(\sqrt{y}),\ \sqrt{y^{10}} = y^5, \text{ so } \sqrt{y^{11}} = y^5\sqrt{y}$$

$$\sqrt{z^{23}} = \left(\sqrt{z^{22}}\right)(\sqrt{z}),\ \sqrt{z^{22}} = z^{11}, \text{ so } \sqrt{z^{23}} = z^{11}\sqrt{z}$$

Step 3: Recombine the simplified expressions. $2x^3y^5z^{11}\sqrt{10xyz}$

Simplify the following square root expressions.

1. $\sqrt{57d^{25}e^{27}f^{22}}$
2. $\sqrt{140h^{26}i^{20}j^9}$
3. $\sqrt{27x^{44}y^{42}z^9}$
4. $\sqrt{75p^{22}q^8r^{21}}$
5. $\sqrt{48k^{47}l^{27}m^3}$
6. $\sqrt{75s^{23}t^7u^{28}}$
7. $\sqrt{63a^8b^{27}c^{42}}$
8. $\sqrt{20p^3q^{44}r^{29}}$
9. $\sqrt{80a^{220}b^{20}c^{27}}$
10. $\sqrt{64m^8n^3p^{22}}$
11. $\sqrt{88r^{27}s^{22}t^{23}}$
12. $\sqrt{40g^{42}h^{25}j^{28}}$
13. $\sqrt{90v^3w^{20}x^{24}}$
14. $\sqrt{50d^7e^9f^{23}}$
15. $\sqrt{45x^{28}y^6z^{23}}$
16. $\sqrt{32a^6b^{23}c^7}$
17. $\sqrt{74j^{24}k^{27}m^7}$
18. $\sqrt{20q^{24}r^{27}s^7}$

Chapter 5 Test

1. $2x^2 + 5x^2 =$

A $10x^4$
B $7x^4$
C $7x^2$
D $10x^2$
E $7x$

2. $-8m^3 + m^3 =$

F $-8m^6$
G $-8m^9$
H $-9m^6$
J $-7m^6$
K $-7m^3$

3. $(6x^3 + x^2 - 5) + (-3x^3 - 2x^2 + 4) =$

A $3x^3 - x^2 - 1$
B $3x^3 - 3x^2 - 1$
C $3x^3 - 3x^2 - 9$
D $-3x^3 - 3x^2 - 1$
E $9x^3 + 3x^2 - 1$

4. $(-7c^2 + 5c + 3) + (-c^2 - 7c + 2) =$

F $-3x^3 - 3x^2 - 1$
G $-8c^2 - 2c + 5$
H $-6c^2 - 12c + 5$
J $-8c^2 - 12c + 5$
K $-6c^2 + 12c + 5$

5. $(5x^3 - 4x^2 + 5) - (-2x^3 - 3x^2) =$

A $3x^3 + x^2 + 5$
B $3x^3 - 7x^2 + 5$
C $7x^3 - x^2 + 5$
D $7x^3 - 7x^2 + 5$
E $3x^3 + 7x^2 + 5$

6. $(-z^3 - 4z^2 - 6) - (3z^3 - 6z + 5) =$

F $-4z^3 - 4z^2 + 6z - 11$
G $-2z^3 - 10z - 1$
H $-4z^3 - 10z^2 - 1$
J $-2z^2 + 2z - 11$
K $-2z^3 + 10z - 1$

7. $(-7d^5)(-3d^2) =$

A $-21d^7$
B $21d^{10}$
C $21d^7$
D $-21d^{10}$
E $-10d^7$

8. $(-5c^3d)(3c^5d^3)(2cd^4) =$

F $30c^{15}d^8$
G $15c^8d^{12}$
H $-17c^{15}d^{12}$
J $-30c^9d^8$
K $-30c^{15}d^{12}$

9. $-11j^2 \times -j^4 =$

A $11j^6$
B $11j^8$
C $-11j^6$
D $-11j^8$
E $11j^{-2}$

10. $-6m^2(7m^2 + 5m - 6) =$

F $-42m^2 + 30m^3 - 36$
G $-42m^4 - 30m^3 + 36m^2$
H $-13m^4 - m^2 + 36m^2$
J $42m^4 - 30m^3 - 36m^2$
K $-42m^4 + 30m^3 + 36m^2$

7.3 Completing the Square

"Completing the Square" is another way of factoring a quadratic equation. To complete the square, convert the equation into a perfect square.

Example 6: Solve $x^2 - 10x + 9 = 0$ by completing the square.

Completing the square:

Step 1: The first step is to get the constant to the other side of the equation. Subtract 9 from both sides:
$x^2 - 10x + 9 - 9 = -9$
$x^2 - 10x = -9$

Step 2: Determine the coefficient of the x. The coefficient in this example is -10. Divide the coefficient by 2 and square the result.
$(-10 \div 2)^2 = (-5)^2 = 25$

Step 3: Add the resulting value, 25, to both sides:
$x^2 - 10x + 25 = -9 + 25$
$x^2 - 10x + 25 = 16$

Step 4: Now factor the $x^2 - 10x + 25$ into a perfect square:
$(x - 5)^2 = 16$

Solving the perfect square:

Step 5: Take the square root of both sides.
$\sqrt{(x-5)^2} = \sqrt{16}$
$(x - 5) = \pm 4$
$(x - 5) = 4$ and $(x - 5) = -4$

Step 6: Solve the two equations.

$(x - 5) = 4$	and	$(x - 5) = -4$
$x - 5 + 5 = 4 + 5$	and	$x - 5 + 5 = -4 + 5$
$x = 9$	and	$x = 1$

Solve for x by completing the square.

1. $x^2 + 2x - 3 = 0$
2. $x^2 - 8x + 7 = 0$
3. $x^2 + 6x - 7 = 0$
4. $x^2 - 16x - 36 = 0$
5. $x^2 - 14x + 49 = 0$
6. $x^2 - 4x = 0$
7. $x^2 + 12x + 27 = 0$
8. $x^2 + 2x - 24 = 0$
9. $x^2 + 12x - 85 = 0$
10. $x^2 - 8x + 15 = 0$
11. $x^2 - 16x + 60 = 0$
12. $x^2 - 8x - 48 = 0$

7.5 Real-World Quadratic Equations

The most common real life situation that would use a quadratic equation is the motion of an object under the force of gravity. Two examples are a ball being kicked into the air or a rocket being shot into the air.

Example 8: A high school football player is practicing his field goal kicks. The equation below represents the height of the ball at a specific time.

$s = -9t^2 + 45t$

$t =$ amount of time in seconds

$s =$ height in feet

Question 1: Where will the ball be at 4 seconds?

Solution 1: Since there are only two variables, you will only need the value of one variable to solve the problem. Simply plug in the number 4 in place of the variable t and solve the equation as shown below.

$s = -9\,(4)^2 + 45\,(4)$

$s = -9\,(16) + 180$

$s = -144 + 180$

$s = 36$

At 4 seconds the ball will be 36 ft in the air.

Question 2: If the ball is 54 ft in the air, how much time has gone by?

Solution 2: This question is similar to the previous one, except that the given variable is different. This time you would be replacing s with 54 and then solve the equation.

$54 = -9t^2 + 45t$ Subtract 54 on both sides.

$0 = -9t^2 + 45t - 54$ Divide the entire equation by -9.

$0 = t^2 - 5t + 6$ Factor the equation.

$0 = (t - 3)\,(t - 2)$ Solve for t.

$t = 3 \qquad t = 2$

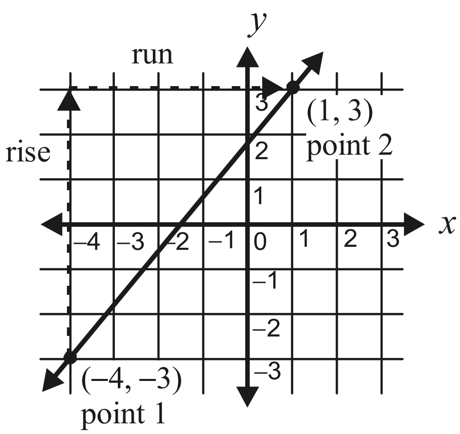
For this question we got 2 answers. The ball is 54 ft in the air when 2 and 3 seconds have gone by.

8.5 Understanding Slope

The slope of a line refers to how steep a line is. Slope is also defined as the rate of change. When we graph a line using ordered pairs, we can easily determine the slope. Slope is often represented by the letter m.

The formula for slope of a line is: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{\text{rise}}{\text{run}}$

Example 6: What is the slope of the following line that passes through the ordered pairs $(-4, -3)$ and $(1, 3)$?



y_2 is 3, the y-coordinate of point 2.

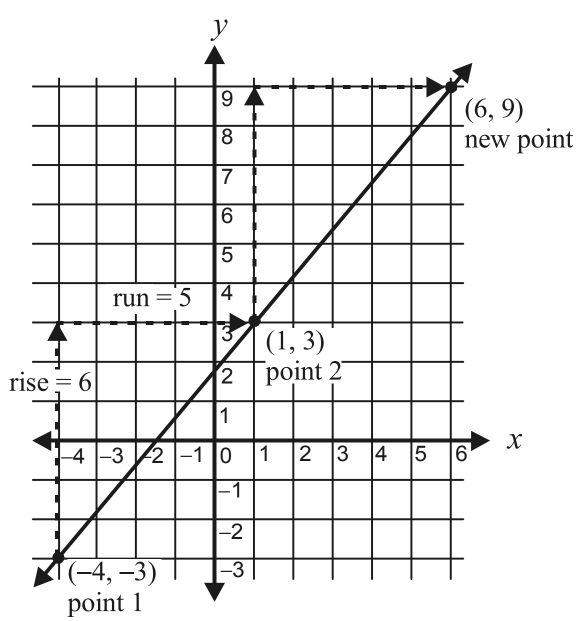
y_1 is -3, the y-coordinate of point 1.

x_2 is 1, the x-coordinate of point 2.

x_1 is -4, the x-coordinate of point 1.

Use the formula for slope given above: $m = \dfrac{3 - (-3)}{1 - (-4)} = \dfrac{6}{5}$

The slope is $\frac{6}{5}$. This shows us that we can go up 6 (rise) and over 5 to the right (run) to find another point on the line.



8.10 Finding the Equation of a Line Using Two Points or a Point and Slope

If you know the slope of a line, and you know the coordinates of one point, you can write the equation for the line. You know the formula for the slope of a line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_2 - y_1}{x_2 - x_1} = m$$

Using algebra, you can see that if you multiply both sides of the equation by $x_2 - x_1$, you get:

$$y - y_1 = m\,(x - x_1) \longleftarrow \text{point-slope form of an equation}$$

Example 17: Write the equation of the line passing through the points $(-2, 3)$ and $(1, 5)$.

Step 1: First, find the slope of the line using the two points given.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{1 - (-2)} = \frac{2}{3}$$

Step 2: Pick one of the two points to use in the point-slope equation. For point $(-2, 3)$, we know $x_1 = -2$ and $y_1 = 3$, and we know $m = \frac{2}{3}$. Substitute these values into the point-slope form of the equation.

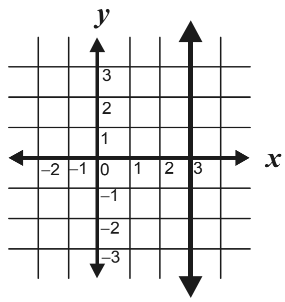
$$y - y_1 = m\,(x - x_1)$$
$$y - 3 = \frac{2}{3}[x - (-2)]$$
$$y - 3 = \tfrac{2}{3}x + \tfrac{4}{3}$$
$$y = \tfrac{2}{3}x + \tfrac{13}{3}$$

Use the point-slope formula to write an equation for each of the following lines.

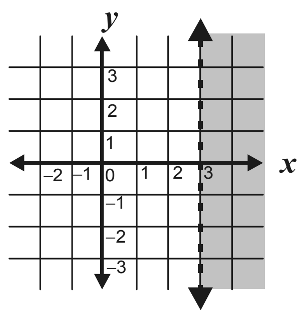
1. $(1, -2)$, $m = 2$
2. $(-3, 3)$, $m = \frac{1}{3}$
3. $(4, 2)$, $m = \frac{1}{4}$
4. $(5, 0)$, $m = 1$
5. $(3, -4)$, $m = \frac{1}{2}$
6. $(-1, -4)$ $(2, -1)$
7. $(2, 1)$ $(-1, -3)$
8. $(-2, 5)$ $(-4, 3)$
9. $(-4, 3)$ $(2, -1)$
10. $(3, 1)$ $(5, 5)$
11. $(-3, 1)$, $m = 2$
12. $(-1, 2)$, $m = \frac{4}{3}$
13. $(2, -5)$, $m = -2$
14. $(-1, 3)$, $m = \frac{1}{3}$
15. $(0, -2)$, $m = -\frac{3}{2}$

8.11 Graphing Inequalities

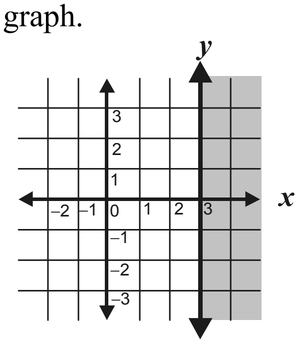
In a previous section, you would graph the equation $x = 3$ as:



In this section, we graph inequalities such as $x > 3$ (read x is greater than 3). To show this, we use a broken line since the points on the line $x = 3$ are not included in the solution. We shade all points greater than 3.



When we graph $x \geq 3$ (read x is greater than or equal to 3), we use a solid line because the points on the line $x = 3$ are included in the graph.



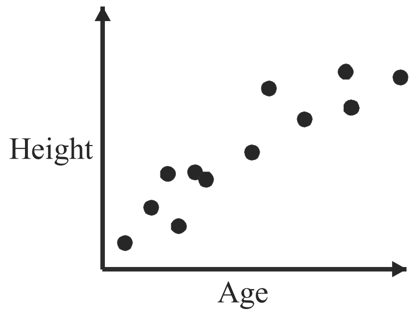
Graph the following inequalities on your own graph paper.

1. $y < 2$
2. $x \geq 4$
3. $y \geq 1$
4. $x < -1$
5. $y \geq -2$
6. $x \leq -4$
7. $x > -3$
8. $y \leq 3$
9. $x \leq 5$
10. $y > -5$
11. $x \geq 3$
12. $y < -1$
13. $x \leq 0$
14. $y > -1$
15. $y \leq 4$
16. $x \geq 0$
17. $y \geq 3$
18. $x < 4$
19. $x \leq -2$
20. $y < -2$
21. $y \geq -4$
22. $x \geq -1$
23. $y \leq 5$
24. $x < -3$

9.11 Interpreting Data in Scatter Plots

You have already learned that scatter plots show the relationship between two variables. Now, you will learn to explain the relationship between variables.

Example 15: The graph below shows the relationship between height and age.

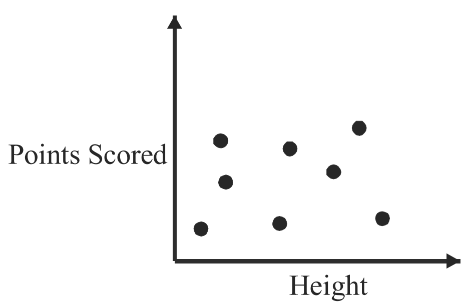


Although it isn't linear, there is clearly a positive relationship between age and height. This means that as age increases, height increases.

Example 16: The graph below shows the relationship between price of an object and the number purchased by customers.

This illustrates a negative relationship. This means as price of an object increases, the number purchased decreases. In other words, if the price of an object goes up, less people will buy that object.

Example 17: The graph below shows the relationship between number of points scored on a test and height



There is no relationship between height and points scored because there is no definable pattern. Therefore, height and number of points scored on a test are not correlated.

Chapter 9 Review

1. Paulo turns on the oven to preheat it. After one minute, the oven temperature is 200°. After 2 minutes, the oven temperature is 325°.

Oven Temperature

Minutes	Temperature
1	200°
2	325°

Assuming the oven temperature rises at a constant rate, write an equation that fits the data.

2. Write an equation that fits the data given below. Assume the data is linear.

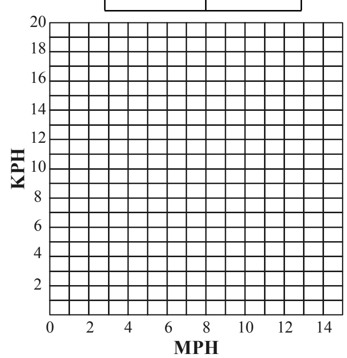
Plumber Charges per Hour

Hour	Charge
1	$170
2	$220

3. The data given below show conversions between miles per hour and kilometers per hour. Based on this data, graph a conversion line on the Cartesian plane below.

Speed

MPH	KPH
5	8
10	16



4. What would be the approximate conversion of 9 mph to kph?

5. What would be the approximate conversion of 13 kph to mph?

6. A bicyclist travels 12 mph downhill. About how many kph is the bicyclist traveling?

7. Use the data given below to graph the interest rate versus the interest rate on $80.00 in one year.

$80.00 Principal

Interest Rate	Interest-1 year
5%	$4.00
10%	$8.00

8. About how much interest would accrue in one year at an 8% interest rate?

9. What is the slope of the line describing interest versus interest rate?

10. What information does the slope give in problem 9?

10.4 Solving Word Problems with Systems of Equations

Certain word problems can be solved using systems of equations.

Example 6: In a game show, Andre earns 6 points for every right answer and loses 12 points for every wrong answer. He has answered correctly 12 times as many as he has missed. His final score was 120. How many times did he answer correctly?

Step 1: Let $r =$ number of right answers. Let $w =$ number of wrong answers.

We know 2 sets of information that can be made into equations with 2 variables.

He earns $+6$ points for right answers and loses 12 points for wrong answers.

$$\begin{aligned} 6r - 12w &= 120 \\ 12w &= r \end{aligned}$$

His wins and losses $= 120$

12 times the number of wrong answers $=$ the number of right answers.

Step 2: Substitute the value for r $(12w)$ in the first equation.

$$\begin{aligned} 6(12w) - 12w &= 120 \\ w &= 2 \end{aligned}$$

Step 3: Substitute the value for w back in the equation.

$$\begin{aligned} 6r - 12(2) &= 120 \\ r &= 24 \end{aligned}$$

Example 7: Ms. Sudberry bought pencils and stickers for her first grade class on two different days. The pencils and stickers cost the same each time she went to the store. How much did she pay for each pencil?

	Pencils	Stickers	Total Cost
Tuesday	30	40	\$47.50
Saturday	60	5	\$20.00

Step 1: Set up your two equations. Let the price of pencils equal x, and the price of stickers equal y.
The amount of the pencils times the price of pencils (x) plus the amount of the stickers times the price of stickers (y) equals the total amount paid for both pencils and stickers.
Equation 1: $30x + 40y = \$47.50$
Equation 2: $60x + 5y = \$20.00$

Step 2: Solve the equations by using one of the methods taught in this chapter. We will use the adding and subtracting method. First, multiply equation 1 by -2, so x will have the same coefficient in each equation but with opposite signs.
$-2(30x + 40y = \$47.50) = -60x - 80y = -\$95.00$

Step 3: Add the new equation 1 to equation 2.

$$\begin{array}{rrcrcr} & -60x & - & 80y & = & -\$95.00 \\ + & 60x & + & 5y & = & \$20.00 \\ \hline & 0x & - & 75y & = & -\$75.00 \end{array}$$

The new equation is $-75y = -\$75.00$.

Step 4: Solve for y.
$-75y = -\$75.00$
$y = \$1.00$
Now, we know the price of stickers, but the question asked for the price of each pencil.

Step 5: Substitute the value of y into either equation and solve for x to find the price of each pencil.
$30x + 40y = \$47.50$
$30x + 40\,(\$1.00) = \$47.50$
$30x + \$40.00 = \$47.50$
$30x = \$7.50$
$x = \$0.25$
The cost of each pencil is $\$0.25$.

Use systems of equations to solve the following word problems.

1. The sum of two numbers is 140 and their difference is 20. What are the two numbers?

2. The sum of two numbers is 126 and their difference is 42. What are the two numbers?

3. Kayla gets paid \$6.00 for raking leaves and \$8.00 for mowing the lawn of each of the neighbors in her subdivision. This year she mowed the lawns 12 times more than she raked leaves. In total, she made \$918.00 for doing both. How many times did she rake the leaves?

4. Prices for the movie are \$4.00 for children and \$8.00 for adults. The total amount of ticket sales is \$1,176. There are 172 tickets sold. How many adults and children buy tickets?

5. A farmer sells a dozen eggs at the market for \$2.00 and one of his bags of grain for \$5.00. He has sold 5 times as many bags of grain as he has dozens of eggs. By the end of the day, he has made \$243.00 worth of sales. How many bags of grain did he sell?

6. Every time Lauren does one of her chores, she gets 15 minutes to talk on the phone. When she does not perform one of her chores, she gets 20 minutes of phone time taken away. This week she has done her chores 5 times more than she has not performed her chores. In total, she has accumulated 165 minutes. How many times has Lauren not performed her chores?

11.9 Independent and Dependent Quantities

As stated previously, a relation is a function if for every element in the domain there is exactly one element in the range. The domain values are generally known, and the range values are determined by solving the function. As each domain value is applied to the function, only one range value will result. The variable that is used to represent the domain values is called the **independent variable** because it is not dependent on any other value. The variable that is used to represent the range values is called the **dependent variable** because its value will be determined by its corresponding domain value.

Example 17: Mrs. Alexander assigned to her students an open book quiz containing 35 questions to be completed at home. Those students who returned the completed quiz by the due date would receive 30 points for turning the assignment in on time and 2 points for each correct answer. A student's grade on the open book quiz can be expressed as the function $f(a) = 30 + 2a$, where a represents the number of correct answers. Identify the independent and dependent variables in this function.

Solution: The independent variable in this problem is a, the number of correct answers because it is not dependent on any other value in the function.
The dependent variable in this problem is the grade, $f(a)$, because it is dependent on the number of correct answers. The dependent variable could have also been assigned a variable such as G, T, or y. Using function notation clearly illustrates in the algebraic sentence that the dependent variable is a function of the independent variable.

Identify the independent and dependent variables in the following functions.

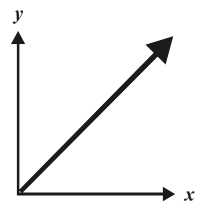
1. A local bookstore is encouraging its customers to drop off used books to be given to schools, libraries, and other community organizations. They are offering to anyone who drops off books a special hard-cover edition of *Oliver Twist* for \$25.95 minus \$0.10 for each used book. The cost for the special edition of *Oliver Twist* can be expressed as $G(u) = \$25.95 - \$0.10u$.

2. Claudia is planning a surprise birthday party for her best friend. To make sure that she has enough food, she is ordering 1 sub sandwich for every person who is coming to the party plus an additional 10 sub sandwiches. The number of sandwiches Claudia is ordering can be written algebraically as follows: $s = 10 + p$.

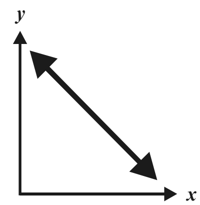
3. John and Mike are brothers who are training for their school swim team. John has been swimming longer than Mike and is able to swim more laps. For every lap that Mike swims, John swims 3, and the number of laps that John swims can be expressed as $j = 3m$.

12.3 Direct and Inverse Variation

The graphs shown below represent functions where x varies with y directly or indirectly. In direct variation, when y increases, the x increases, and when y decreases, x decreases. In inverse variation, also called indirect variation, when y increases, x decreases, and when y decreases, x increases.



Direct Variation



Inverse Variation

Example 3: Direct and indirect variation can be demonstrated with function tables.

Table 1

x	y
0	3
1	4
2	7
3	12
4	19

Table 2

x	y
0	20
1	18
2	16
3	14
4	12

Notice in Table 1, as x increases, y increases also. This means that function Table 1 represents a direct variation between x and y. On the other hand, Table 2 shows a decrease in y when x increases. This means that function Table 2 represents an indirect variation between x and y.

Direct variation occurs in a function when y varies directly, or in the same way, as x varies. The two values vary by a proportional factor, k. The variation is treated just like a proportion.

Example 4: If y varies directly with x, and $y = 18$ when $x = 12$, what is the value of y when $x = 6$?

Step 1: Set up the values in a proportion like you did in the previous two sections. Be sure to put the correct corresponding x and y values on the same line within the fractions.

$$\underline{x \text{ values}} \qquad \underline{y \text{ values}}$$

$$\frac{12}{6} = \frac{18}{y}$$

Step 2: Solve for y by multiplying the diagonals together and setting them equal to one another.

$12 \times y = 6 \times 18$

$\frac{12y}{12} = \frac{108}{12}$ Divide both sides by 12.

$y = 9$

Chapter 12 Review

Solve the following proportions.

1. $\frac{8}{x} = \frac{1}{2}$

2. $\frac{2}{5} = \frac{x}{10}$

3. $\frac{x}{6} = \frac{3}{9}$

4. $\frac{4}{9} = \frac{8}{x}$

Look at the tables below and determine whether the pattern is linear or quadratic. What is the missing term?

5.

x	-2	-1	0	1	2	3
y	14	11	10	11	14	?

6.

x	4	5	6	7	8	9
y	-1	0	1	2	3	?

Find the pattern for the following number sequences, and then find the nth number requested.

7. 0, 1, 2, 3, 4 pattern_______

8. 0, 1, 2, 3, 4 20th number_______

9. 1, 3, 5, 7, 9 pattern_______

10. 1, 3, 5, 7, 9 25th number_______

11. 3, 6, 9, 12, 15 pattern_______

12. 3, 6, 9, 12, 15 30th number_______

13. If $y = 12$ and $x = 6$, using inverse variation, what is the value of y when $x = 20$?

14. If $y = 10$ and $x = 5$, what is the value of y when $x = 4$? Use direct variation to solve.

15. Sandra waters 30 plants in 10 minutes. At this same rate, how many plants can she water in 25 minutes?

Justin receives a bill from his internet service provider. The first four months of service are charged according to the table below:

	January	February	March	April
Hours	0	10	5	25
Charge	\$4.95	\$14.45	\$9.70	\$28.70

16. Write a formula for the cost of n hours of internet service.

17. What is the greatest number of hours he can get on the internet and still keep his bill under \$20.00?

Lisa is baking cookies for the Fall Festival. She bakes 27 cookies with each batch of batter. However, she has a defective oven, which results in 5 cookies in each batch being burnt.

18. Write a formula for the number of cookies available for the festival as a result of Lisa baking n batches of cookies.

19. How many batches does she need in order to produce 300 cookies for the festival?

Practice Test 1

You can use the formula sheet on page viii as needed.

1. Water hyacinths were introduced into the swamps of Louisiana to put oxygen back into the water. The hyacinths reproduced rapidly and soon became a nuisance. Read the growth table below, and then answer the question that follows.

Number of Days	Number of Hyacinths
1	2
21	4
41	8
61	16

Assuming the pattern continues, how many hyacinths will there be at 121 days?

A 16
B 32
C 64
D 128
E 256

A.1.E

2.

$x =$ **Number of sneezes per day**	$y =$ **Number of tissues used per day**
6	2
15	8
18	10
600	398

Yuri always catches colds during the winter. This table represents the average number of tissues he uses in a day, a function of the number of times he sneezes. Which of the following is the formula for his tissue use?

F $y = \frac{2}{3}x - 2$
G $y = \frac{3}{2}x - 7$
H $y = \frac{2}{3}x - 7$
J $y = \frac{3}{2}x - 2$
K $y = 2x - \frac{3}{2}$

A.1.D

3. The functional relationship between altitude (A) above sea level (in feet) and the approximate boiling point (B) of water (in degrees Fahrenheit) may be expressed by the equation $B = -0.00176A + 212$. What is the approximate boiling point of water at $2,500$ feet?

A 194.4

B 207.6

C 208.3

D 216.4

E 212.1

A.1.C

4. A scientist learns to tell the temperature outside by the number of chirps a cricket makes per minute. He uses the formula:

$$t = \frac{n}{8} + 5$$

where $t =$ the outdoor temperature and $n =$ the number of chirps made by the cricket in 1 minute. What is the dependent variable in the above formula?

F the outdoor temperature

G the number of chirps made by the cricket in 1 minute

H the number of crickets

J the number of chirps made by the cricket in 1 hour

K the indoor temperature

A.1.A

Practice Test 2

You can use the formula sheet on page viii as needed.

1. Shawna owns Busy Canines, a dog walking business. The chart shows how much she charges for her dog walking services.

Charges for Dog Walking

# of Minutes (m)	Total Charges (c)
1	\$12.50
5	\$14.50
10	\$17.00
15	\$19.50
20	\$22.00
25	\$24.50

Which equation represents the data given?

A $m = \frac{1}{2}c + 12$

B $c = \frac{1}{2}m + 12$

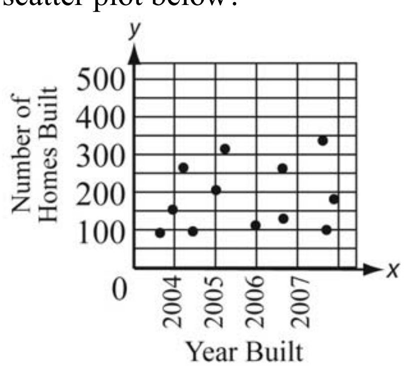
C $c = 12m + 0.50$

D $m = \frac{1}{2}c$

E $c = \frac{1}{2}m + 12.50$

A.1.D

2. What type of relationship is depicted in the scatter plot below?



F quadratic
G nonlinear
H cubic
J linear
K none of the above

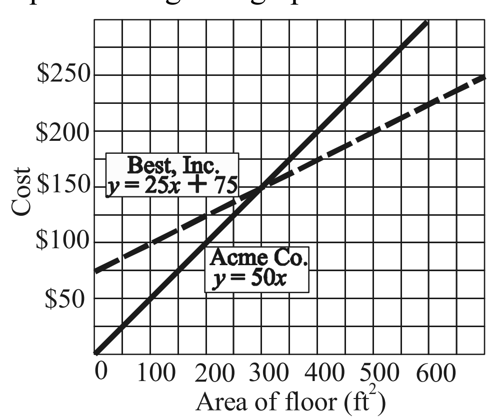
A.1.B

3. Aja works for an air sanitizing company selling their products at a home improvement store. She makes \$12 an hour plus \$20 for every product she sells. She works forty hours a week. If she were to write a function expressing the amount of pay she receives from her place of employment each week, what would the independent variable be?

A the number of products she sells
B the amount of money she makes in total
C the forty hours a week she works
D the amount of money she makes each hour
E Aja

A.1.A

4. Idean is comparing the cost of two floor waxing services. Acme Co. charges \$50 per 100 square feet. Best, Inc. charges \$25 per 100 square feet plus a \$75 fee. Two equations representing price as a function of square footage are graphed below.



Acme Co. is less expensive than Best, Inc. for

F all size floors.
G exactly 300 ft^2.
H areas more than 300 ft^2.
J areas more than 250 ft^2.
K areas less than 300 ft^2.

A.1.E