American Book Company's

Passing the North Carolina 8th Grade End-Of-Grade Test in Mathematics

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Contents

Diagnostic Test

1. Which expression is not an irrational number?

A $\sqrt{10}$
B $\sqrt{2}$
C $\frac{10}{50}$
D π

1.01

2. Which expression is an irrational number?

A 12
B $-\frac{20}{6}$
C $\sqrt{3}$
D 0.5

1.01

3. Mr. Fields found the distance between the school and the park to be $\sqrt{6560}$ ft. What is the approximate distance?

A 42 ft
B 62 ft
C 102 ft
D 81 ft

1.01

4. Ned is using the following formula to determine how many hours, (h) he will need to study for all of his exams next week.

$h = 2.5s + 10$

Using the equation given above, how many hours will Ned need to study if he has six subjects (s)?

A 10
B 15
C 25
D 30

5.04

5. Find the solutions for $x^2 - 9 = 0$

A $(0, 3)$
B $(-3, 3)$
C $(0, -3)$
D $(1, -3)$

5.04

6. Find the perimeter of a rectangle with a length of $\left(4 + \sqrt{6}\right)$ ft and a width of $\left(2 + \sqrt{6}\right)$ ft.

A $12 + 4\sqrt{6}$ ft
B $12 + \sqrt{6}$ ft
C $12\sqrt{6}$ ft
D $8\sqrt{6}$ ft

1.01

7. Which group of numbers is expressed from least to greatest?

A $\sqrt{5}$, $\frac{5}{10}$, 0.05
B 0.99, $\frac{4}{10}$, $\sqrt{3}$
C $\sqrt{7}$, $\frac{2}{10}$, 1
D $\frac{6}{10}$, 1, $\sqrt{8}$

1.01

8. Which group of numbers is expressed from greatest to least?

A $\frac{3}{4}$, $\sqrt{2}$, 4
B $\sqrt{9}$, 2, 0.98
C $\sqrt{2}$, 4, 0.80
D 1.01, $\sqrt{2}$, 3

1.01

1.13 Comparing the Relative Magnitude of Numbers

When comparing the relative magnitude of numbers, the greater than ($>$), less than ($<$), and the equal to ($=$) signs are the ones most frequently used. The simplest way to compare numbers that are in different notations, like percent, decimals, and fractions, is to change all of them to one notation. Decimals are the easiest to compare.

Example 14: Which is larger: $1\frac{1}{4}$ or 1.3?

Step 1: Change $1\frac{1}{4}$ to a decimal. $\frac{1}{4} = 0.25$, so $1\frac{1}{4} = 1.25$.

Step 2: Compare the two values in decimal form.
$1.25 < 1.3$, so 1.3 is the larger of the two values.

Example 15: Which is smaller: 60% or $\frac{2}{3}$?

Step 1: Change both values to decimals.
$60\% = 0.6$ and $\frac{2}{3} = 0.\overline{66}$

Step 2: Compare the two values in decimal form.
0.6 is smaller than $0.\overline{66}$, so $60\% < \frac{2}{3}$.

Fill in each box with the correct sign.

1. $23.4 \square 23\frac{1}{2}$
2. $17\% \square 0.17$
3. $\frac{3}{8} \square 37.5\%$
4. $25\% \square \frac{2}{10}$
5. $234\% \square 23.4$
6. $\frac{1}{7} \square 14\%$
7. $13.95 \square 13\frac{8}{9}$
8. $4.0 \square 40\%$
9. $25\% \square \frac{3}{2}$
10. $\frac{12}{4} \square 300\%$
11. $6\% \square \frac{1}{16}$
12. $1.\overline{33} \square \frac{4}{3}$
13. $0.8 \square \frac{4}{5}$
14. $75\% \square \frac{3}{4}$
15. $\frac{5}{8} \square 62\%$

Compare the sums, differences, products, and quotients below. Fill in each box with the correct sign.

16. $(32 + 15) \square (65 - 17)$
17. $(45 - 13) \square (31 + 9)$
18. $(24 \div 4) \square (24 \div 6)$
19. $(48 \div 6) \square (4 \times 3)$
20. $(4 \times 3) \square (48 \div 6)$
21. $(18 \times 4) \square (5 \times 17)$
22. $[(1 + 3) + 5] \square [5 + (3 + 1)]$
23. $[1 + (3 + 5)] \square [(5 - 3) + 1]$
24. $(25 \div 5) \square (5 \times 5)$
25. $(6 + 4 \div 2) \square [(6 + 4) \div 2]$

2.7 Cube Roots

Cube roots look like square roots, except that there is a "3" raised in front of the root sign:

Square root of 64: $\sqrt{64}$
Cube root of 64: $\sqrt[3]{64}$

In fact, they function very much like square roots, with one important difference. Recall asking, "What is the square root of 64?" means:

"What number multiplied by itself equals 64?"

Asking "What is the cube root of 64?" means:

"What number multiplied 3 times ('cubed') by itself equals 64?"

The answer is 4. $4 \times 4 \times 4 = 64$.

Find the cube root of the following numbers.

Examples: $\sqrt[3]{27}$ $\quad 3 \times 3 \times 3 = 27$ so $\sqrt[3]{27} = 3$

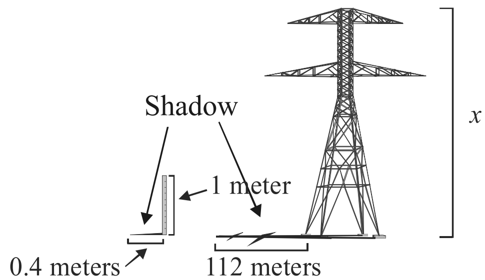
$\sqrt[3]{1000}$ $\quad 10 \times 10 \times 10 = 1000$ so $\sqrt[3]{1000} = 10$

Find the cube roots of the following numbers.

1. $\sqrt[3]{1}$
2. $\sqrt[3]{8}$
3. $\sqrt[3]{64}$
4. $\sqrt[3]{125}$
5. $\sqrt[3]{27}$
6. $\sqrt[3]{\frac{64}{27}}$
7. $\sqrt[3]{1000}$
8. $\sqrt[3]{\frac{125}{1000}}$

4.3 Proportion Word Problems

Example 3: A stick one meter long is held perpendicular to the ground and casts a shadow 0.4 meters long. At the same time, an electrical tower casts a shadow 112 meters long. Use ratio and proportion to find the height of the tower.



Step 1: Set up a proportion using the numbers in the problem. Put the shadow lengths on one side of the equation and put the heights on the other side. The 1 meter height is paired with the 0.4 meter length, so let them both be top numbers. Let the unknown height be x.

shadow length		object height
$\frac{0.4}{112}$	$=$	$\frac{1}{x}$

Step 2: Solve the proportion as you did on page 50.

$112 \times 1 = 112$ $\qquad$ $112 \div 0.4 = 280$

Answer: The tower height is 280 meters.

Use ratio and proportion to solve the following problems.

1. Rudolph can mow a lawn that measures $1,000$ square feet in 2 hours. At that rate, how long would it take him to mow a lawn $3,500$ square feet?
2. Faye wants to know how tall her school building is. On a sunny day, she measures the shadow of the building to be 6 feet. At the same time she measures the shadow cast by a 5-foot statue to be 2 feet. How tall is her school building?
3. Out of every 5 students surveyed, 2 listen to country music. At that rate, how many students in a school of 800 listen to country music?
4. Butterfly, a Labrador retriever, has a litter of 8 puppies. Four are black. At that rate, how many puppies in a litter of 10 would be black?
5. According to the instructions on a bag of fertilizer, 5 pounds of fertilizer are needed for every 100 square feet of lawn. How many square feet will a 25-pound bag cover?
6. A race car can travel 2 laps in 5 minutes. At this rate, how long will it take the race car to complete 100 laps ?
7. If it takes 7 cups of flour to make 4 loaves of bread, how many loaves of bread can you make from 35 cups of flour?
8. If 3 pounds of jelly beans cost $6.30, how much would 2 pounds cost?
9. For the first 4 home football games, the concession stand sold a total of 600 hotdogs. If that ratio stays constant, how many hotdogs will sell for all 10 home games?

6.12 Real-World Linear Equations

Linear equations are very useful mathematical tools. They allow us to show relationships between two variables.

Example 16: A local cell phone company uses the equation $y = \frac{5}{2}x + 10$ to determine the charges for usage where $y =$ the cost and $x =$ the minutes used. How much will Jessica's bill be if she talked for 40 minutes?

Step 1: Substitute the known value in for x.
$y = \frac{5}{2}(40) + 10$

Step 2: Simplify.
$y = 100 + 10 = 110$
Jessica's bill will be $\$110$.

Example 17: Vincent bought a luxury car for $\$165,000$ and its value has depreciated linearly. After 5 years the value was $\$137,000$. What is the amount of yearly depreciation?

Step 1: First find how much the car's value depreciated in 5 years.
$\$165,000 - \$137,000 = \$28,000$

Step 2: Next, find the yearly depreciation by dividing $\$28,000$ by the amount of years, 5.
$\$28,000 \div 5 = \$5,600$
The value of Vincent's car depreciated $\$5,600$ each year.

Example 18: In 1990, the average cost of a new house was $\$123,000$. By the year 2000, the average cost of a new house was $\$134,150$. Based on a linear model, what is the predicted average cost for 2008?

Step 1: First, we need to find the difference between the average cost of a new house in the year 1990 and the average cost of a new house in the year 2000.
$\$134,150 - \$123,000 = \$11,150$

Step 2: Next, we need to find how much the average cost of a new house went up each year. Since it had been 10 years, divide the difference between the value in 2000 and 1990 by 10.
$\$11,150 \div 10 = \$1,115$

Step 3: Multiply the amount the average cost of a new house went up each year by the number of years between 2000 and 2008.
$\$1,115 \times 8 = \$8,920$

Step 4: Lastly, add the average cost of a new house in the year 2000 with the amount found in step 3.
$\$134,150 + \$8,920 = 143,070$
$\$143,070$ is the predicted average cost of a new house for 2008.

7.5 Slope-Intercept Form of a Line

An equation that contains two variables, each to the first degree, is a linear equation. The graph for a linear equation is a straight line. To put a linear equation in slope-intercept form, solve the equation for y. This form of the equation shows the slope and the y-intercept. Slope-intercept form follows the pattern of $y = mx + b$. The "m" represents slope, and the "b" represents the y-intercept. The y-intercept is the point at which the line crosses the y-axis.

When the slope of a line is not 0, the graph of the equation shows a direct variation between y and x. When y increases, x increases in a certain proportion. The proportion stays constant. The constant is called the slope of the line.

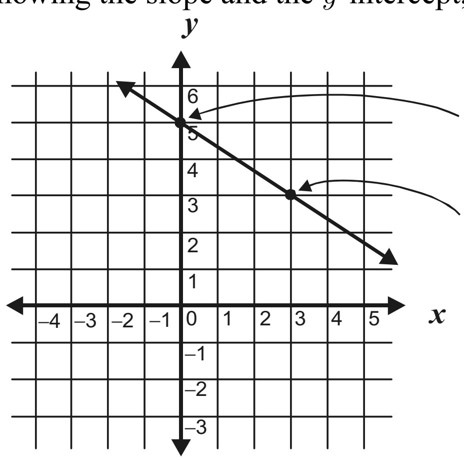
Example 10: Put the equation $2x + 3y = 15$ in slope-intercept form. What is the slope of the line? What is the y-intercept? Graph the line.

Step 1: Solve for y:

$$\begin{array}{ccccccc} 2x & + & 3y & = & 15 & & \\ -2x & & & & -2x & & \\ \hline & & \frac{3y}{3} & = & -\frac{2x}{3} & + & \frac{15}{3} \end{array}$$

slope-intercept form: $y = -\frac{2}{3}x + 5$ The slope is $-\frac{2}{3}$ and the y-intercept is 5.

Step 2: Knowing the slope and the y-intercept, we can graph the line.



The y-intercept is 5, so the line passes through the point $(0, 5)$ on the y-axis.

The slope is $-\frac{2}{3}$, so go down 2 and over 3 to get a second point.

Put each of the following equations in slope-intercept form by solving for y. On your graph paper, graph the line using the slope and y-intercept.

1. $4x - y = 5$
2. $2x + 4y = 16$
3. $3x - 2y = 10$
4. $x + 3y = -12$
5. $6x + 2y = 0$
6. $8x - 5y = 10$
7. $-2x + y = 4$
8. $-4x + 3y = 12$
9. $-6x + 2y = 12$
10. $x - 5y = 5$
11. $3x - 2y = -6$
12. $3x + 4y = 2$
13. $-x = 2 + 4y$
14. $2x = 4y - 2$
15. $6x - 3y = 9$
16. $4x + 2y = 8$
17. $6x - y = 4$
18. $-2x - 4y = 8$
19. $5x + 4y = 16$
20. $6 = 2y - 3x$

9.4 The Line of Best Fit

At this point, you now understand how to plot points on a Cartesian plane. You also understand how to find the data trend on a Cartesian plane. These skills are necessary to accomplish the next task, determining the line of best fit. The line of best fit is a straight line that demonstrates the relationship between two variables. The line does not necessarily divide the plotted points into two areas, but this sometimes is the best way to estimate the line of best fit.

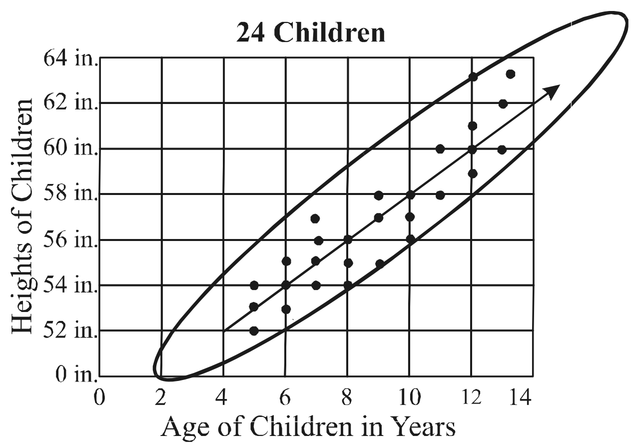
To estimate the line of best fit, you must first draw a scatter plot of all data points. Once this is accomplished, draw an oval around all of the points plotted. Draw a line through the points in such a way that the line separates half the points from one another. You may now use this line to answer questions.

Example 5: The following data set contains the heights of children between 5 and 13 years old. Make a scatter plot and draw the line of best fit to represent the trend. Using the graph, determine the height for a 14-year old child.

Age 5: 4’6",4’4",4’5"
Age 6: 4’7", 4’5", 4’6"
Age 7: 4’9", 4’7", 4’6", 4’8"
Age 8: 4’8", 4’6", 4’7"
Age 9: 4’9", 4’7", 4’10"
Age 10: 4’9", 4’8", 4’ 10"
Age 11: 5’0", 4’10"
Age 12: 5’1", 4’11", 5’0", 5’3"
Age 13: 5’3", 5’2", 5’0", 5’1"

In this example, the data points lay in a positive sloping direction. To determine the line of best fit, all data points were circled, then a line of best fit was drawn. Half of the points lay below, half above the line of best fit drawn bisecting the narrow length of the oval.

To find the height of a 14–year old, simply continue the line of best fit forward. In this case, the height is 62 inches.



Plot the data sets below, then draw the line of best fit. Next, use the line to estimate the value of the next measurement.

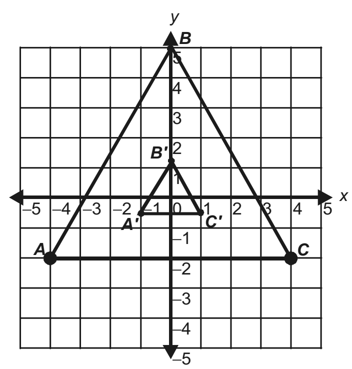
1. Selected values of the Sleekster Brand Light Compact Vehicles: New Vehicle: $\$13,000$.
 1 year old: $\$12,000$, $\$11,000$, $\$12,500$
 2 year old: $\$9,000$, $\$10,500$, $\$9,500$
 3 year old: $\$8,500$, $\$8,000$, $\$9,000$
 4 year old: $\$7,500$, $\$6,500$, $\$6,000$
 5 year old: ?

2. The relationship between string length and kite height for the following kites:
 (L = 500 ft, H = 400 ft) (L = 250 ft, H = 150ft) (L = 100 ft, H = 75 ft)
 (L = 500ft, H = 350 ft) (L = 250 ft, H = 200 ft) (L = 100 ft, H = 50 ft)
 (L = 600 ft, H = ?)

10.12 Dilations

A **dilation** of a geometric figure is either an enlargement or a reduction of the figure. The point at which the figure is either reduced or enlarged is called the center of dilation. The dilation of a figure is always the product of the original and a **scale factor**. The scale factor is always a positive number that is multiplied by the coordinates of a shape's vertices, which is usually illustrated in a coordinate plane. If the scale factor is greater than one, then the resulting dilated figure will be an enlargement of the original figure. If the scale factor is less than one, then the resulting dilated figure will be a reduction of the original figure.

Example 12: The triangle *ABC* has been dilated by a scale factor of $\frac{1}{4}$.



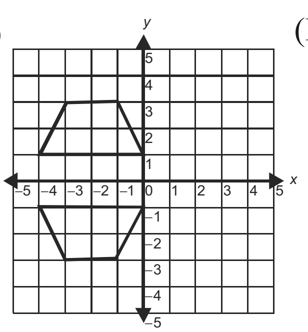
The first step in finding the dilated object is to list all the vertices of the original object, *ABC*. The next step is to multiply each of the coordinates of the vertices of *ABC* by the scale factor, $\frac{1}{4}$, to find the coordinates of the dilated figure. Lastly, draw the dilated object on the coordinate plane as shown above.

$A:(-4,-2)$ $A':\left(-1,-\frac{1}{2}\right)$
$B:(0,5)$ $B':\left(0,\frac{5}{4}\right)$
$C:(4,-2)$ $C':\left(1,-\frac{1}{2}\right)$

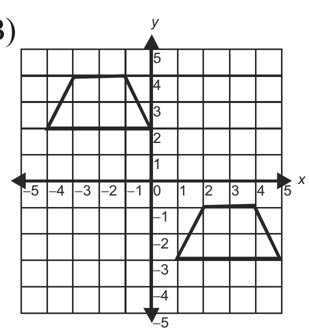
Note: Since the scale factor is less than one, the dilated figure *A′B′C′* is a reduction of original triangle, *ABC*.

Circle the coordinate plane that contains the shape that has been dilated.

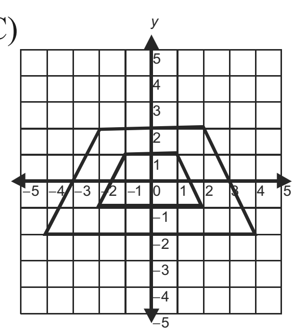
(A)



(B)



(C)



Practice Test 1

1. Which statement is true?

   A π is an irrational number, because it is a non-repeating decimal

   B -4 is an irrational number, because it is a negative integer

   C $\frac{1}{4}$ is a rational number, because it is a repeating decimal

   D $\sqrt{2}$ is a rational number, because it is a positive integer

   1.01

2. Which set of numbers contains only rational numbers?

   A $-\frac{4}{3}, \sqrt{6}, 1.002$

   B $\sqrt{7}, 2, -\frac{1}{4}$

   C $\frac{1}{2}, -6.4, \sqrt{25}$

   D $3\pi, 16\frac{1}{2}, \sqrt{36}$

   1.01

3. Joey's yard is shaped like a square and has an area of 960 sq ft. What is the approximate length of each side?

   A 28 ft
   B 40 ft
   C 31 ft
   D 48 ft

   1.01

4. Which group of numbers satisfies the following equation $x \leq 3$?

   A $\sqrt{3}, \frac{2}{3}, -2$

   B $\pi + 1, 0.54, \frac{1}{4}$

   C $2^2, \sqrt{9}, 3$

   D $-4, \frac{3}{4}, 3.1$

   1.01

5. Which set of numbers is NOT ordered from least to greatest?

   A $-4, \sqrt{2}, 2$

   B $\frac{1}{4}, \pi, 2^2$

   C $\frac{7}{8}, 1, \frac{6}{10}$

   D $0.15, 1.5, 1.05$

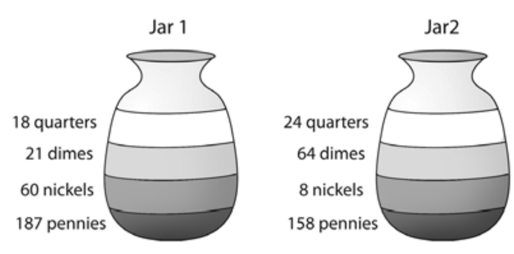
   1.01

6. Josh's garden has an area of 150 sq ft. If his garden is shaped like a square, what is the approximate length of each side?

   A 600 sq ft
   B 15 sq ft
   C 12 sq ft
   D 9 sq ft

   1.01

7. Principal Baker of Ace Middle School agreed to give $\$30.00$ to the school's PTA. If he donates the change he has in both jars, how much more will Principal Baker need to contribute?



   A $\$3.15$
   B $\$5.15$
   C $\$4.15$
   D $\$4.25$

   1.02

Practice Test 2

1. If the area of square $PQRS$ is 500 sq ft, the length of each side is in between which two numbers?

   A 30 and 31
   B 22 and 23
   C 24 and 25
   D 27 and 28

   1.01

2. Which box(es) contains the most rational numbers?

Box P	Box Q	Box R	Box S
π, $\sqrt{17}$, $-\frac{3}{5}$	$-\sqrt{4}$, 2^2, 6	$\frac{5}{6}$, $\sqrt{49}$, 1.015	$\sqrt{16}$, $\sqrt{4}$, $\sqrt{3}$

   A Box P
   B Boxes Q and R
   C Boxes R and S
   D Box R

   1.01

3. The number $\dfrac{22}{7}$ is a(n)

   A natural number.
   B integer.
   C rational number.
   D irrational number.

   1.01

4. Which number is the largest?

   A 0.21
   B 1.60
   C $\sqrt{3}$
   D $\frac{8}{10}$

   1.01

5. The length of Melanie's pencil measures 5 inches. Which irrational number could possibly be the length of this pencil?

   A $\sqrt{5}$
   B $\sqrt{10}$
   C $\sqrt{13}$
   D $\sqrt{26}$

   1.01

6. Linda works 3 days per week at $\$6.00$ per hour. If she works 4 hours per day, how long will it take Linda to earn $\$350$?

   A 4 weeks
   B 5 weeks
   C 6 weeks
   D 8 weeks

   1.02

7. Carson is starting his own lawn mowing business. The lawn equipment costs $\$1,207$. He plans to charge $\$25.00$ per lawn. How many lawns must he mow to earn a profit?

   A 49
   B 48
   C 47
   D 50

   1.02

8. Betty bought 12 donuts for $\$6.72$. What is the average cost per donut *before* taxes?

   A $\$6.53$
   B $\$0.53$
   C $\$6.40$
   D $\$0.56$

   1.02