



PASSING THE INDIANA  
END-OF-COURSE ASSESSMENT  
IN  
ALGEBRA I

ERICA DAY  
COLLEEN PINTOZZI

---

AMERICAN BOOK COMPANY  
P. O. BOX 2638  
WOODSTOCK, GEORGIA 30188-1383  
TOLL FREE 1 (888) 264-5877    TOLL FREE FAX 1 (866) 827-3240  
WEB SITE: [www.americanbookcompany.com](http://www.americanbookcompany.com)

# Contents

<b>Acknowledgements</b>	<b>ii</b>
<b>Preface</b>	<b>viii</b>
<b>Diagnostic Test</b>	<b>1</b>
Session 1	1
Session 2	5
<b>1 Exponents</b>	<b>10</b>
1.1 Understanding Exponents	10
1.2 Multiplying Exponents with the Same Base	11
1.3 Multiplying Fractional Exponents with the Same Base	11
1.4 Multiplying Exponents Raised to an Exponent	12
1.5 Expressions Raised to a Power	12
1.6 Fractions Raised to a Power	13
1.7 More Multiplying Exponents	13
1.8 Negative Exponents	14
1.9 Multiplying with Negative Exponents	14
1.10 Dividing with Exponents	15
1.11 Order of Operations	16
Chapter 1 Review	18
Chapter 1 Test	19
<b>2 Roots</b>	<b>21</b>
2.1 Square Root	21
2.2 Simplifying Square Roots Using Factors	21
2.3 Adding, Subtracting, and Simplifying Square Roots	22
2.4 Multiplying and Simplifying Square Roots	23
2.5 Dividing and Simplifying Square Roots	24
2.6 Cube Roots	25
2.7 Rational Exponents	26
2.8 Comparing Real Number Expressions	27
Chapter 2 Review	28
Chapter 2 Test	29
<b>3 Solving Multi-Step Linear Equations and Inequalities</b>	<b>31</b>
3.1 Two-Step Linear Equations	31

3.2	Two-Step Linear Equations with Fractions	32
3.3	More Two-Step Linear Equations with Fractions	33
3.4	Combining Like Terms	34
3.5	Solving Linear Equations with Like Terms	34
3.6	Solving for a Variable	36
3.7	Removing Parentheses	37
3.8	Multi-Step Linear Equations	38
3.9	Solving Linear Inequalities Using Solution Sets	40
3.10	Multi-Step Linear Inequalities	41
3.11	Combined Linear Inequalities	43
	Chapter 3 Review	44
	Chapter 3 Test	45
<b>4</b>	<b>Algebra Word Problems</b>	<b>46</b>
4.1	Rate	46
4.2	More Rates	47
4.3	Dimensional Analysis	48
4.4	Algebra Word Problems	50
4.5	Real-World Linear Equations	51
4.6	Word Problems with Formulas	53
4.7	Age Problems	54
4.8	Consecutive Integer Problems	56
4.9	Inequality Word Problems	57
	Chapter 4 Review	58
	Chapter 4 Test	59
<b>5</b>	<b>Polynomials</b>	<b>61</b>
5.1	Properties of Addition and Multiplication	61
5.2	Adding and Subtracting Monomials	62
5.3	Adding Polynomials	63
5.4	Subtracting Polynomials	64
5.5	Multiplying Monomials	66
5.6	Multiplying Monomials with Different Variables	67
5.7	Dividing Monomials	68
5.8	Extracting Monomial Roots	69
5.9	Monomial Roots with Remainders	70
5.10	Multiplying Monomials by Polynomials	71

## Contents

5.11	Dividing Polynomials by Monomials	72
5.12	Removing Parentheses and Simplifying	73
5.13	Multiplying Two Binomials	74
	Chapter 5 Review	76
	Chapter 5 Test	77
<b>6</b>	<b>Factoring</b>	<b>79</b>
6.1	Finding the Greatest Common Factor of Polynomials	79
6.2	Finding the Numbers	82
6.3	More Finding the Numbers	83
6.4	Factoring Trinomials	84
6.5	More Factoring Trinomials	86
6.6	Factoring More Trinomials	87
6.7	Factoring the Difference of Two Squares	89
6.8	Simplifying Algebraic Ratios	91
6.9	Solving Algebraic Proportions	92
	Chapter 6 Review	93
	Chapter 6 Test	94
<b>7</b>	<b>Solving Quadratic Equations</b>	<b>95</b>
7.1	Solving Perfect Squares	97
7.2	Completing the Square	98
7.3	Proof of the Quadratic Formula	99
7.4	Using the Quadratic Formula	100
7.5	Real-World Quadratic Equations	101
7.6	Solving Equations That Contain Radical Expressions	104
	Chapter 7 Review	105
	Chapter 7 Test	106
<b>8</b>	<b>Graphing and Writing Equations and Inequalities</b>	<b>107</b>
8.1	Graphing Linear Equations	107
8.2	Graphing Horizontal and Vertical Lines	109
8.3	Finding the Intercepts of a Line	110
8.4	Understanding Slope	111
8.5	Slope-Intercept Form of a Line	113
8.6	Graphing a Line Knowing a Point and Slope	115
8.7	Finding the Equation of a Line Using Two Points or a Point and Slope	116
8.8	Graphing Inequalities	117

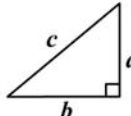
Chapter 8 Review	120
Chapter 8 Test	121
<b>9 Applications of Graphs</b>	<b>124</b>
9.1 Changing the Slope or Y-Intercept of a Line	124
9.2 Equations of Parallel Lines	126
9.3 Equations of Perpendicular Lines	126
9.4 Graphing Quadratic Equations	129
9.5 Finding the Intercepts of a Quadratic Equation	130
9.6 Graphing Basic Functions	132
9.7 Writing an Equation From Data	134
9.8 Graphing Linear Data	135
Chapter 9 Review	137
Chapter 9 Test	139
<b>10 Pairs of Linear Equations and Inequalities</b>	<b>141</b>
10.1 Pairs of Linear Equations	141
10.2 Finding Common Solutions for Intersecting Lines	143
10.3 Solving Pairs of Equations by Substitution	144
10.4 Solving Pairs of Equations by Adding or Subtracting	146
10.5 Graphing Pairs of Inequalities	148
10.6 Solving Word Problems with Pairs of Equations	149
Chapter 10 Review	152
Chapter 10 Test	153
<b>11 Relations and Functions</b>	<b>156</b>
11.1 Relations	156
11.2 Determining Domain and Range From Graphs	158
11.3 Functions	160
11.4 Recognizing Functions	161
11.5 Interpreting a Graph Representing a Given Situation	164
Chapter 11 Review	167
Chapter 11 Test	169
<b>12 Mathematical Reasoning</b>	<b>171</b>
12.1 Mathematical Reasoning/Logic	171
12.2 Deductive and Inductive Arguments	173
12.3 Logic	175

## Contents

12.4	Using Diagrams to Solve Problems	177
12.5	Trial and Error Problems	178
12.6	Pattern Problems	179
12.7	Reasonable Solutions	181
	Chapter 12 Review	183
	Chapter 12 Test	185
	<b>Practice Test 1</b>	<b>187</b>
	Session 1	187
	Session 2	191
	<b>Practice Test 2</b>	<b>194</b>
	Session 1	194
	Session 2	198
	<b>Index</b>	<b>202</b>

# End-of-Course Assessment Algebra I Reference Sheet

**Pythagorean Theorem**



$a^2 + b^2 = c^2$

**Distance Formula**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$d$  = distance between points 1 and 2

**Midpoint Formula**

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$M$  = point halfway between points 1 and 2

**Standard Form of a Linear Equation**

$$Ax + By = C$$

(where  $A$  and  $B$  are not both zero)

**Standard Form of a Quadratic Equation**

$$ax^2 + bx + c = 0$$

(where  $a \neq 0$ )

**Quadratic Formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(where  $ax^2 + bx + c = 0$  and  $a \neq 0$ )

**Equation of a Line**

**Slope-Intercept Form:**  $y = mx + b$   
 where  $m$  = slope and  $b$  =  $y$ -intercept

**Point-Slope Form:**  $y - y_1 = m(x - x_1)$

**Simple Interest Formula**

$$I = prt$$











where  $I$  = interest  
 $p$  = principal  
 $r$  = rate  
 $t$  = time

**Slope of a Line**

Let  $(x^1, y^1)$  and  $(x^2, y^2)$  be two points in a plane

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y^2 - y^1}{x^2 - x^1}$$

(where  $x_2 \neq x_1$ )

Shape	Formulas for Area (A) and Circumference (C)	
Triangle 	$A = \frac{1}{2}bh = \frac{1}{2} \times \text{base} \times \text{height}$	
Trapezoid 	$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2} \times \text{sum of bases} \times \text{height}$	
Parallelogram 	$A = bh = \text{base} \times \text{height}$	
Circle 	$A = \pi r^2 = \pi \times \text{square of radius}$	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
	$C = 2\pi r = 2 \times \pi \times \text{radius}$	
Figure	Formulas for Volume (V) and Surface Area (SA)	
Cube 	$SA = 6s^2 = 6 \times \text{length of side squared}$	
Cylinder (total) 	$SA = 2\pi rh + 2\pi r^2$	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
	$SA = 2 \times \pi \times \text{radius} \times \text{height} + 2 \times \pi \times \text{radius squared}$	
Sphere 	$SA = 4\pi r^2 = 4 \times \pi \times \text{radius squared}$ $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times \text{radius cubed}$	
Cone 	$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times \text{radius squared} \times \text{height}$	
Pyramid 	$V = \frac{1}{3}Bh = \frac{1}{3} \times \text{area of base} \times \text{height}$	
Prism 	$V = Bh = \text{area of base} \times \text{height}$	

# Diagnostic Test

## Session 1

1. Simplify:  $\frac{x^2 - 4}{x^2 + x - 6}$

- A  $\frac{x - 2}{x + 3}$
- B  $\frac{x + 2}{x + 3}$
- C  $\frac{x + 2}{x - 2}$
- D  $\frac{x - 2}{x - 3}$

A1.7.1

2. A kindergarten teacher needs to buy 10 markers for each student to color with. She has 19 kids in her class. If markers come in packs of 6, what is the minimum number of packs she must buy?

- A 32
- B 31
- C 3
- D 2

A1.2.6

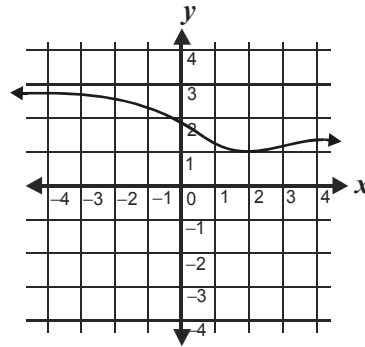
3. Simplify the following expression.

$$\frac{x^2 - 4}{x^2 + 5x + 6}$$

- A 1
- B  $\frac{x + 2}{x + 3}$
- C  $\frac{x - 2}{x + 2}$
- D  $\frac{x - 2}{x + 3}$

A1.7.1

4. Is the following graph a function?



- A Yes, because it passes the vertical line test.
- B Yes, because it passes the horizontal line test.
- C No, because it fails the vertical line test.
- D No, because it fails the horizontal line test.

A1.3.3

5. Solve for  $b$ .  $4b + a = 13$

- A  $13 - 4a$
- B  $13 - 4b$
- C  $\frac{13 - a}{4}$
- D  $\frac{13 - b}{a}$

A1.2.2

6. Solve the proportion.

$$\frac{2x - 1}{3} = \frac{x + 4}{6}$$

- A  $x = 6$
- B  $x = 3$
- C  $x = 2$
- D  $x = 1$

A1.7.2

## 1.10 Dividing with Exponents

Exponents that have the same base can also be divided.

**Example 24:**  $\frac{3^5}{3^3}$  This problem means  $3^5 \div 3^3$ . Let us look at 2 ways to solve this problem.

**Solution 1:**  $\frac{3^5}{3^3} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3} = 3 \cdot 3 = 9$  First, rewrite the fraction with the exponents in expanded form, and then multiply.

**Solution 2:**  $\frac{3^5}{3^3} = 3^{5-3} = 3^2 = 9$  A quick way to simplify this same problem is to subtract the exponents. **When dividing exponents with the same base, subtract the exponents.**

**Example 25:**  $\frac{(4x)^{-3}}{2x^4}$

**Step 1:**  $(4x)^{-3} = \frac{1}{(4x)^3} = \frac{1}{4^3x^3}$  Remove the parentheses from the top of the fraction.

**Step 2:**  $\frac{1}{4^3x^3 \cdot 2x^4} = \frac{1}{128x^7}$  The bottom of the fraction remains the same, so put the two together and simplify.

**Simplify the problems below. You may be able to cancel. Be sure to follow order of operations. Remove parentheses before canceling.**

1.  $\frac{5^5}{5^3}$

7.  $\frac{(7^2)^3}{7^5}$

13.  $\frac{x^3}{(x^2)^3}$

19.  $\frac{12^{-4}}{12^{-2}}$

2.  $\frac{x^2}{x^3}$

8.  $\frac{(x^3)^4}{x^6}$

14.  $\frac{2^2}{2^7}$

20.  $\frac{6^{12}}{6^9}$

3.  $\frac{(10^2)^4}{10^5}$

9.  $\frac{4^3}{4^2}$

15.  $\frac{6^2}{6}$

21.  $\frac{8^8}{8^{10}}$

4.  $\frac{3^5}{3^2}$

10.  $\frac{2}{(2^2)^2}$

16.  $\frac{9^{11}}{9^9}$

22.  $\frac{3(x^{-3})^{-2}}{3x^7}$

5.  $\frac{8^{10}}{8^8}$

11.  $\frac{(3x)^{-2}}{9x^5}$

17.  $\frac{(15)^5}{15^6}$

23.  $\frac{7^3}{7^5}$

6.  $\frac{5^2}{5}$

12.  $\frac{(11^4)^4}{(11^7)^2}$

18.  $\frac{(x^3)^{-2}}{(x^2)^5}$

24.  $\frac{10^3}{10^{-1}}$

From the given set and inequality, find the solution set.

1.  $5x > 100$  in  $\{0, 10, 20, 30, 40\}$

7.  $1 - x < 5$  in  $\{-97, -60, 97, -1, -70, -47\}$

2.  $7x - 10 < 30$  in  $\{4, 5, 6, 7\}$

8.  $-7x - 7 < -10$  in  $\{-45, 21, 49, -12, 8, 45\}$

3.  $-10x + 5 < 0$  in  $\{-1, 0, 1, 2\}$

9.  $5x - 8 < -4$  in  $\{-63, -47, -48, -99, 11, 76\}$

4.  $3x + 7 > 28$  in  $\{5, 6, 7, 8, 9, 10\}$

10.  $-x - 2 < 3$  in  $\{-1, -2, -2, 1, -5, 0\}$

5.  $-x - 10 > 0$  in  $\{-30, -20, -10, 0, 10, 20\}$

11.  $3x + 4 < -4$  in  $\{-5, -4, -3, -2, -1, 0\}$

6.  $\frac{1}{5}x > 5$  in  $\{-5, 0, 10, 25, 45\}$

12.  $-x - 4 < 2$  in  $\{26, -29, 1, -19, -26, -5\}$

### 3.10 Multi-Step Linear Inequalities

Remember that adding and subtracting with inequalities follow the same rules as equations. When you multiply or divide both sides of an inequality by the same positive number, the rules are the same as for equations. However, when you multiply or divide both sides of an inequality by a **negative** number, you must **reverse** the inequality symbol.

**Example 16:**

$$\begin{aligned} -x &> 4 \\ (-1)(-x) &< (-1)(4) \\ x &< -4 \end{aligned}$$

**Example 17:**

$$\begin{aligned} -4x &< 2 \\ \frac{-4x}{-4} &> \frac{2}{-4} \\ x &> -\frac{1}{2} \end{aligned}$$

Reverse the symbol when you multiply or divide by a negative number.

When solving multi-step inequalities, first add and subtract to isolate the term with the variable. Then multiply and divide.

**Example 18:**

$$2x - 8 > 4x + 1$$

**Step 1:** Add 8 to both sides.

$$\begin{aligned} 2x - 8 + 8 &> 4x + 1 + 8 \\ 2x &> 4x + 9 \end{aligned}$$

**Step 2:** Subtract  $4x$  from both sides.

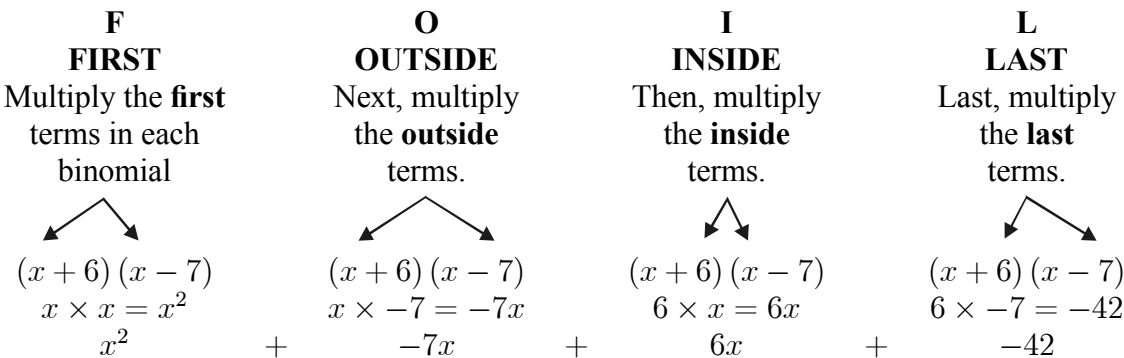
$$\begin{aligned} 2x - 4x &> 4x + 9 - 4x \\ -2x &> 9 \end{aligned}$$

**Step 3:** Divide by  $-2$ . Remember to change the direction of the inequality sign.

$$\begin{aligned} \frac{-2x}{-2} &< \frac{9}{-2} \\ x &< -\frac{9}{2} \end{aligned}$$

### 5.13 Multiplying Two Binomials

When you multiply two binomials such as  $(x + 6)(x - 7)$ , you must multiply each term in the first binomial by each term in the second binomial. The easiest way is to use the **FOIL** method. If you can remember the word **FOIL**, it can help you keep order when you multiply. The "F" stands for **first**, "O" stands for **outside**, "I" stands for **inside**, and "L" stands for **last**.



Now just combine like terms,  $6x - 7x = -x$ , and write your answer.

$$(x + 6)(x - 7) = x^2 - x - 42.$$

**Note:** It is customary for mathematicians to write polynomials in descending order. That means that the term with the highest-number exponent comes first in a polynomial. The next highest exponent is second and so on. When you use the **FOIL** method, the terms will always be in the customary order. You just need to combine like terms and write your answer.

**Multiply the following binomials.**

1.  $(y - 8)(y + 3)$

6.  $(9v - 4)(3v + 5)$

11.  $(7t + 3)(4t - 2)$

2.  $(4x + 5)(x + 20)$

7.  $(20p + 4)(5p + 3)$

12.  $(5y - 20)(5y + 20)$

3.  $(5b - 3)(3b - 5)$

8.  $(3h - 20)(-4h - 7)$

13.  $(a + 6)(3a + 7)$

4.  $(6g + 4)(g - 20)$

9.  $(w - 5)(w - 8)$

14.  $(3z - 9)(z - 5)$

5.  $(8k - 7)(-5k - 3)$

10.  $(6x + 2)(x - 4)$

15.  $(7c + 4)(6c + 7)$

## 6.7 Factoring the Difference of Two Squares

Let's give an example of a **perfect square**.

25 is a perfect square because  $5 \times 5 = 25$

49 is a perfect square because  $7 \times 7 = 49$

Any variable with an even exponent is a perfect square.

$y^2$  is a perfect square because  $y \times y = y^2$

$y^4$  is a perfect square because  $y^2 \times y^2 = y^4$

When two terms that are both perfect squares are subtracted, factoring those terms is very easy. To factor the difference of perfect squares, you use the square root of each term, a plus sign in the first factor, and a minus sign in the second factor.

**Example 11:**

Factor  $4x^2 - 9$

This example has two terms which are both perfect squares, and the terms are subtracted.

**Step 1:**

$(2x \quad 3)(2x \quad 3)$

Find the square root of each term.

Use the square roots in each of the factors.

**Step 2:**

$(2x + 3)(2x - 3)$

Use a plus sign in one factor and a minus sign in the other factor.

**Check:**

Multiply to check.  $(2x + 3)(2x - 3) = 4x^2 - 6x + 6x - 9 = 4x^2 - 9$

The inner and outer terms add to zero.

**Example 12:**

Factor  $81y^4 - 1$

**Step 1:**

$(9y^2 + 1)(9y^2 - 1)$

Factor like the example above.

Notice, the second factor is also the difference of two perfect squares.

**Step 2:**

$(9y^2 + 1)(3y + 1)(3y - 1)$

Factor the second term further.

**Note: You cannot factor the sum of two perfect squares.**

**Check:**

Multiply in reverse to check your answer.

$$\begin{aligned} (9y^2 + 1)(3y + 1)(3y - 1) &= (9y^2 + 1)(9y^2 - 3y + 3y - 1) = \\ (9y^2 + 1)(9y^2 - 1) &= 81y^4 + 9y^2 - 9y^2 - 1 = 81y^4 - 1 \end{aligned}$$

## 7.5 Real-World Quadratic Equations

The most common real life situation that would use a quadratic equation is the motion of an object under the force of gravity. Two examples are a ball being kicked into the air or a rocket being shot into the air.

**Example 7:** A high school football player is practicing his field goal kicks. The equation below represents the height of the ball at a specific time.

$$s = -9t^2 + 45t$$

$t$  = amount of time in seconds

$s$  = height in feet

**Question 1:** Where will the ball be at 4 seconds?

**Solution 1:** Since there are only two variables, you will only need the value of one variable to solve the problem. Simply plug in the number 4 in place of the variable  $t$  and solve the equation as shown below.

$$s = -9(4)^2 + 45(4)$$

$$s = -9(16) + 180$$

$$s = -144 + 180$$

$$s = 36$$

At 4 seconds the ball will be 36 ft in the air.

**Question 2:** If the ball is 54 ft in the air, how much time has gone by?

**Solution 2:** This question is similar to the previous one, except that the given variable is different. This time you would be replacing  $s$  with 54 and then solve the equation.

$$54 = -9t^2 + 45t \quad \text{Subtract 54 on both sides.}$$

$$0 = -9t^2 + 45t - 54 \quad \text{Divide the entire equation by } -9.$$

$$0 = t^2 - 5t + 6 \quad \text{Factor the equation.}$$

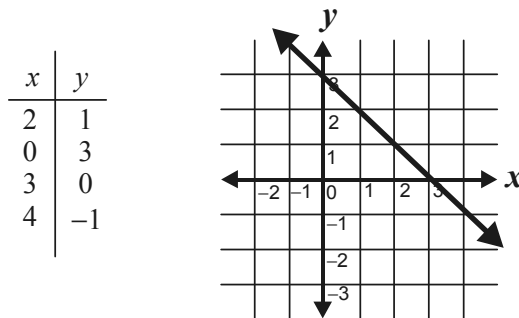
$$0 = (t - 3)(t - 2) \quad \text{Solve for } t.$$

$$t = 3 \quad t = 2$$

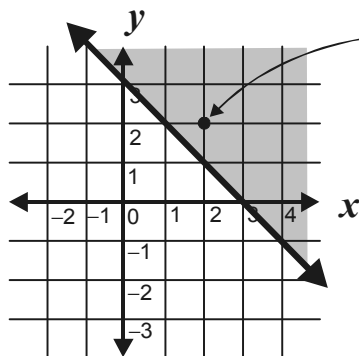
For this question we got 2 answers. The ball is 54 ft in the air when 2 and 3 seconds have gone by.

**Example 14:** Graph  $x + y \geq 3$ .

**Step 1:** First, we graph  $x + y \geq 3$  by changing the inequality to an equality. Think of ordered pairs that will satisfy the equation  $x + y = 3$ . Then, plot the points, and draw the line. As shown below, this line divides the Cartesian plane into 2 half-planes,  $x + y \geq 3$  and  $x + y \leq 3$ . One half-plane is above the line, and the other is below the line.



**Step 2:** To determine which side of the line to shade, first choose a test point. If the point you choose makes the inequality true, then the point is on the side you shade. If the point you choose does not make the inequality true, then shade the side that does not contain the test point.



For our test point, let's choose  $(2, 2)$ .  
Substitute  $(2, 2)$  into the inequality.

$$x + y \geq 3$$

$$2 + 2 \geq 3$$

$4 \geq 3$  is true, so shade the side that includes this point.

Use a solid line because of the  $\geq$  sign.

**Graph the following inequalities on your own graph paper.**

1.  $x + y \leq 4$

5.  $x - y \geq -2$

9.  $x \geq y + 2$

2.  $x + y \geq 3$

6.  $x < y + 4$

10.  $x < -y + 1$

3.  $x \geq 5 - y$

7.  $x + y < -1$

11.  $-x + y > 1$

4.  $x \leq 1 + y$

8.  $x - y \leq 0$

12.  $-x - y < -2$

## 10.6 Solving Word Problems with Pairs of Equations

Certain word problems can be solved using systems of equations.

**Example 7:** In a game show, Andre earns 6 points for every right answer and loses 12 points for every wrong answer. He has answered correctly 12 times as many as he has missed. His final score was 120. How many times did he answer correctly?

**Step 1:** Let  $r$  = number of right answers. Let  $w$  = number of wrong answers.  
We know 2 sets of information that can be made into equations with 2 variables.  
He earns +6 points for right answers and loses 12 points for wrong answers.

$$\begin{array}{r} \swarrow \qquad \qquad \qquad \searrow \\ 6r - 12w = 120 \\ 12w = r \end{array}$$

His wins and losses = 120

12 times the number of wrong answers = the number of right answers.

**Step 2:** Substitute the value for  $r$  ( $12w$ ) in the first equation.

$$\begin{array}{r} 6(12w) - 12w = 120 \\ w = 2 \end{array}$$

**Step 3:** Substitute the value for  $w$  back in the equation.

$$\begin{array}{r} 6r - 12(2) = 120 \\ r = 24 \end{array}$$

**Example 8:** Ms. Sudberry bought pencils and stickers for her first grade class on two different days. The pencils and stickers cost the same each time she went to the store. How much did she pay for each pencil?

	Pencils	Stickers	Total Cost
Tuesday	30	40	\$47.50
Saturday	60	5	\$20.00

**Step 1:** Set up your two equations. Let the price of pencils equal  $x$ , and the price of stickers equal  $y$ .  
The amount of the pencils times the price of pencils ( $x$ ) plus the amount of the stickers times the price of stickers ( $y$ ) equals the total amount paid for both pencils and stickers.

Equation 1:  $30x + 40y = \$47.50$

Equation 2:  $60x + 5y = \$20.00$

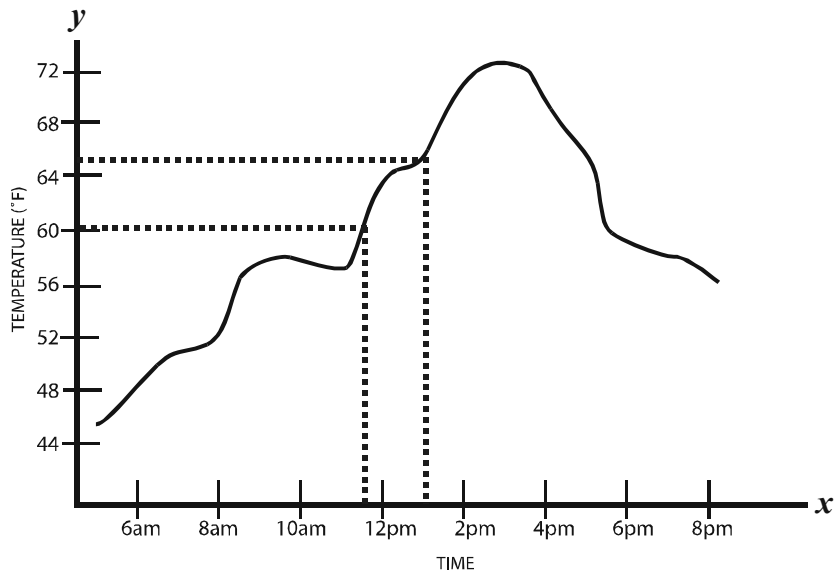
**Step 2:** Solve the equations by using one of the methods taught in this chapter. We will use the adding and subtracting method. First, multiply equation 1 by  $-2$ , so  $x$  will have the same coefficient in each equation but with opposite signs.

$$-2(30x + 40y = \$47.50) = -60x - 80y = -\$95.00$$

### 11.5 Interpreting a Graph Representing a Given Situation

Real-world situations are sometimes modeled by graphs. Although an equation cannot be written for most of these graphs, interpreting these graphs provides valuable information. Situations may be represented on a graph as a function of time, length, temperature, etc.

The graph below depicts the temperature of a pond at different times of the day. Refer to the graph as you read through examples 1 and 2.



**Example 8:** If it is known that a specific breed of fish is most active in waters between  $60^{\circ}\text{F}$  and  $65^{\circ}\text{F}$ , what time of the day would this fish be the most active in this particular pond?

To find the answer, draw lines from the  $60^{\circ}\text{F}$  and  $65^{\circ}\text{F}$  points on the  $y$ -axis to the graph. Then, draw vertical lines from the graph to the  $x$ -axis. The time range between the two vertical lines on the  $x$ -axis indicates the time that the fish are most active. It can be determined from the graph that the fish are most active between 11:30 am and 1:00 PM.

**Example 9:** Describe the way the temperature of the pond acts as a function of time.

At 6:00 AM, the temperature of the pond is about  $47^{\circ}\text{F}$ . The temperature increases relatively steadily throughout the morning and early afternoon. The temperature peaks at  $72^{\circ}\text{F}$ , which is around 2:30 PM during the day. Afterwards, the temperature of the pond starts to decrease. The later it gets in the evening, the more the temperature of the water decreases. The graph shows that at 8 PM the temperature of the pond is about  $57^{\circ}\text{F}$ .

## 12.3 Logic

A **conditional statement** is a type of logical statement that has two parts, a **hypothesis** and a **conclusion**. The statement is written in "if-then" form, where the "if" part contains the hypothesis and the "then" part contains the conclusion. For example, let's start with the statement "Two lines intersect at exactly one point." We can rewrite this as a conditional statement in "if-then" form as follows:

$$\underbrace{\text{If two lines intersect}}_{\text{hypothesis}}, \text{ then } \underbrace{\text{their intersection is at exactly one point.}}_{\text{conclusion}}$$

Conditional statements may be true or false. To show that a statement is false, you need only to provide a single **counterexample** which shows that the statement is not always true. To show that a statement is true, on the other hand, you must show that the conclusion is true for all occasions in which the hypothesis occurs. This is often much more difficult.

**Example 3:** Provide a counterexample to show that the following conditional statement is false:

If  $x^2 = 4$ , then  $x = 2$ .

To begin with, let  $x = -2$ .

The hypothesis is true, because  $(-2)^2 = 4$ .

For  $x = -2$ , however, the conclusion is false even though the hypothesis is true. Therefore, we have provided a counterexample to show that the conditional statement is false.

The **converse** of a conditional statement is an "if-then" statement written by switching the hypothesis and the conclusion. For example, for the conditional statement "If a figure is a quadrilateral, then it is a rectangle," the converse is "If a figure is a rectangle, then it is a quadrilateral."

The **inverse** of a conditional statement is written by negating the hypothesis and conclusion of the original "if-then" conditional statement. Negating means to change the meaning so it is the negative, or opposite, of its original meaning. The inverse of the conditional statement "If a figure is a quadrilateral, then it is a rectangle" is "If a figure is **not** a quadrilateral, then it is **not** a rectangle."

The **contrapositive** of a conditional statement is written by negating the converse. That is, switch the hypothesis and conclusion of the original statement, and make them both negative. The contrapositive of the conditional statement "If a figure is a quadrilateral, then it is a rectangle" is "If a figure is not a rectangle, then it is not a quadrilateral."

# Practice Test 1

## Session 1

1. Simplify  $\frac{x+7}{7x^2-343}$ .

- A  $\frac{1}{2}$
- B  $\frac{1}{7(x^2-343)}$
- C  $\frac{1}{7(x+7)}$
- D  $\frac{1}{7(x-7)}$

A1.7.1

2. Solve the proportion  $\frac{3x+2}{2} = \frac{3x-12}{6}$ .

- A  $x = 1$
- B  $x = -1$
- C  $x = -3$
- D  $x = -6$

A1.7.2

3. Use the substitution method to solve the pair of equations  $y = 3x + 11$  and  $y = 2x$ .

- A  $(-1, -2)$
- B  $(1, 2)$
- C  $(-11, -22)$
- D  $(11, 22)$

A1.5.3

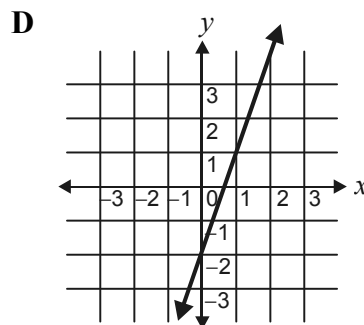
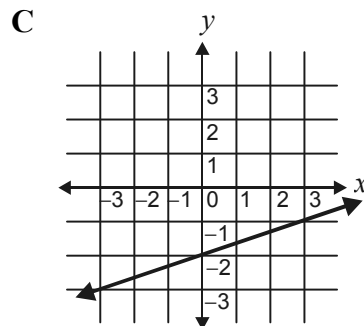
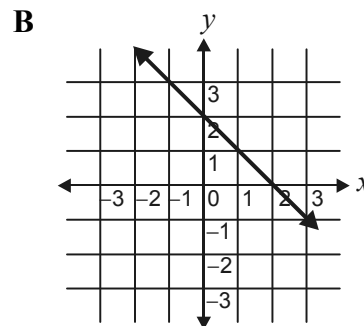
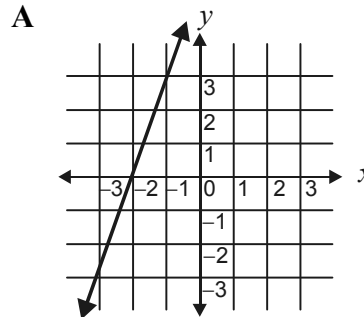
4. Which property is demonstrated below?

$$(a \times b) \times c = a \times (b \times c)$$

- A Associative Property of Multiplication
- B Commutative Property of Multiplication
- C Identity Property of Multiplication
- D Inverse Property of Multiplication

A1.1.3

5. Which is the graph of  $x - 3y = 6$ ?



A1.4.1

# Practice Test 2

## Session 1

1. Use addition to solve the equations:

$$x - y = 1 \text{ and } x + y = 6.$$

- A  $(\frac{7}{4}, \frac{3}{4})$
- B  $(\frac{10}{3}, \frac{8}{3})$
- C  $(\frac{5}{2}, \frac{7}{2})$
- D  $(\frac{7}{2}, \frac{5}{2})$

A1.5.4

2. Solve the algebraic proportion

$$\frac{x + 2}{7} = \frac{x + 4}{21}.$$

- A  $x = -1$
- B  $x = 2$
- C  $x = -4$
- D  $x = 7$

A1.7.2

3. What is the domain of  $y = x^3 - 7$ ?

- A  $-7 \leq x < \infty$
- B  $-\infty < x < \infty$
- C  $-\infty < y \leq -7$
- D  $-7 \leq y < \infty$

A1.3.4

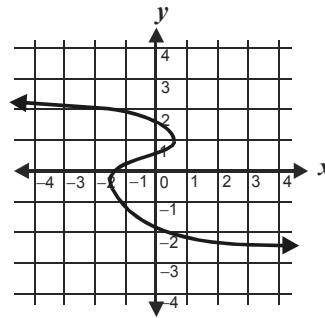
4. What is the range of the function

$$y = -x^2 + 1?$$

- A  $-\infty < y < \infty$
- B  $-\infty < y \leq 0$
- C  $-\infty < y \leq 1$
- D  $1 \leq y < \infty$

A1.3.4

5. Is the graph below a function?



- A Yes, because it passes the vertical line test.
- B Yes, because it passes the horizontal line test.
- C No, because it fails the vertical line test.
- D No, because it fails the horizontal line test.

A1.3.3

6. What is 790 centimeters per second in kilometers per hour?

- A 28.44 km/hr
- B 28,440 km/hr
- C 0.2844 km/hr
- D 0.0079

A1.1.5

7. What is the slope of a line represented by the equation  $x = 2y - 3$ ?

- A  $-2$
- B  $-\frac{1}{2}$
- C  $\frac{1}{2}$
- D  $2$

A1.4.2