American Book Company's

PASSING THE INDIANA END-OF-COURSE ASSESSMENT IN ALGEBRA I

ERICA DAY

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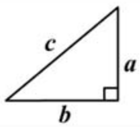
End-of-Course Assessment Algebra I Reference Sheet

Pythagorean Theorem



$a^2 + b^2 = c^2$

Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

d = distance between points 1 and 2

Midpoint Formula

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

M = point halfway between points 1 and 2

Standard Form of a Linear Equation

$Ax + By = C$

(where A and B are not both zero)

Standard Form of a Quadratic Equation

$ax^2 + bx + c = 0$

(where $a \neq 0$)

Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(where $ax^2 + bx + c = 0$ and $a \neq 0$)

Equation of a Line

Slope-Intercept Form: $y = mx + b$
where m = slope and b = y-intercept

Point-Slope Form: $y - y_1 = m(x - x_1)$

Simple Interest Formula

$I = prt$

where I = interest

p = principal

r = rate

t = time

Slope of a Line

Let (x^1, y^1) and (x^2, y^2) be two points in a plane

slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{y^2 - y^1}{x^2 - x^1}$

(where $x_2 \neq x_1$)

Shape	Formulas for Area (A) and Circumference (C)	
Triangle	$A = \frac{1}{2}bh = \frac{1}{2} \times$ base $\times$ height	
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2} \times$ sum of bases $\times$ height	
Parallelogram	$A = bh$ = base $\times$ height	
Circle	$A = \pi r^2 = \pi \times$ square of radius $C = 2\pi r = 2 \times \pi \times$ radius	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Figure	**Formulas for Volume (V) and Surface Area (SA)**	
Cube	$SA = 6s^2 = 6 \times$ length of side squared	
Cylinder (total)	$SA = 2\pi rh + 2\pi r^2$ $SA = 2 \times \pi \times$ radius $\times$ height $+ 2 \times \pi \times$ radius squared	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Sphere	$SA = 4\pi r^2 = 4 \times \pi \times$ radius squared $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times$ radius cubed	
Cone	$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times$ radius squared $\times$ height	
Pyramid	$V = \frac{1}{3}Bh = \frac{1}{3} \times$ area of base $\times$ height	
Prism	$V = Bh$ = area of base $\times$ height	

Diagnostic Test

Session 1

1. Simplify: $\dfrac{x^2-4}{x^2+x-6}$

A $\dfrac{x-2}{x+3}$

B $\dfrac{x+2}{x+3}$

C $\dfrac{x+2}{x-2}$

D $\dfrac{x-2}{x-3}$

A1.7.1

2. A kindergarten teacher needs to buy 10 markers for each student to color with. She has 19 kids in her class. If markers come in packs of 6, what is the minimum number of packs she must buy?

A 32
B 31
C 3
D 2

A1.2.6

3. Simplify the following expression.

$$\frac{x^2-4}{x^2+5x+6}$$

A 1

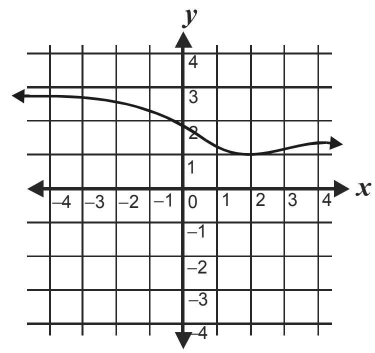
B $\dfrac{x+2}{x+3}$

C $\dfrac{x-2}{x+2}$

D $\dfrac{x-2}{x+3}$

A1.7.1

4. Is the following graph a function?



A Yes, because it passes the vertical line test.
B Yes, because it passes the horizontal line test.
C No, because it fails the vertical line test.
D No, because it fails the horizontal line test.

A1.3.3

5. Solve for b. $4b+a=13$

A $13-4a$

B $13-4b$

C $\dfrac{13-a}{4}$

D $\dfrac{13-b}{a}$

A1.2.2

6. Solve the proportion.

$$\frac{2x-1}{3}=\frac{x+4}{6}$$

A $x=6$
B $x=3$
C $x=2$
D $x=1$

A1.7.2

1.10 Dividing with Exponents

Exponents that have the same base can also be divided.

Example 24: $\frac{3^5}{3^3}$ This problem means $3^5 \div 3^3$. Let us look at 2 ways to solve this problem.

Solution 1: $\frac{3^5}{3^3} = \frac{\not{3} \cdot \not{3} \cdot \not{3} \cdot 3 \cdot 3}{\not{3} \cdot \not{3} \cdot \not{3}} = 3 \cdot 3 = 9$ First, rewrite the fraction with the exponents in expanded form, and then multiply.

Solution 2: $\frac{3^5}{3^3} = 3^{5-3} = 3^2 = 9$ A quick way to simplify this same problem is to subtract the exponents. **When dividing exponents with the same base, subtract the exponents.**

Example 25: $\frac{(4x)^{-3}}{2x^4}$

Step 1: $(4x)^{-3} = \frac{1}{(4x)^3} = \frac{1}{4^3x^3}$ Remove the parentheses from the top of the fraction.

Step 2: $\frac{1}{4^3x^3 \cdot 2x^4} = \frac{1}{128x^7}$ The bottom of the fraction remains the same, so put the two together and simplify.

Simplify the problems below. You may be able to cancel. Be sure to follow order of operations. Remove parentheses before canceling.

1. $\frac{5^5}{5^3}$
2. $\frac{x^2}{x^3}$
3. $\frac{(10^2)^4}{10^5}$
4. $\frac{3^5}{3^2}$
5. $\frac{8^{10}}{8^8}$
6. $\frac{5^2}{5}$
7. $\frac{(7^2)^3}{7^5}$
8. $\frac{(x^3)^4}{x^6}$
9. $\frac{4^3}{4^2}$
10. $\frac{2}{(2^2)^2}$
11. $\frac{(3x)^{-2}}{9x^5}$
12. $\frac{(11^4)^4}{(11^7)^2}$
13. $\frac{x^3}{(x^2)^3}$
14. $\frac{2^2}{2^7}$
15. $\frac{6^2}{6}$
16. $\frac{9^{11}}{9^9}$
17. $\frac{(15)^5}{15^6}$
18. $\frac{(x^3)^{-2}}{(x^2)^5}$
19. $\frac{12^{-4}}{12^{-2}}$
20. $\frac{6^{12}}{6^9}$
21. $\frac{8^8}{8^{10}}$
22. $\frac{3(x^{-3})^{-2}}{3x^7}$
23. $\frac{7^3}{7^5}$
24. $\frac{10^3}{10^{-1}}$

From the given set and inequality, find the solution set.

1. $5x > 100$ in $\{0, 10, 20, 30, 40\}$
2. $7x - 10 < 30$ in $\{4, 5, 6, 7\}$
3. $-10x + 5 < 0$ in $\{-1, 0, 1, 2\}$
4. $3x + 7 > 28$ in $\{5, 6, 7, 8, 9, 10\}$
5. $-x - 10 > 0$ in $\{-30, -20, -10, 0, 10, 20\}$
6. $\frac{1}{5}x > 5$ in $\{-5, 0, 10, 25, 45\}$
7. $1 - x < 5$ in $\{-97, -60, 97, -1, -70, -47\}$
8. $-7x - 7 < -10$ in $\{-45, 21, 49, -12, 8, 45\}$
9. $5x - 8 < -4$ in $\{-63, -47, -48, -99, 11, 76\}$
10. $-x - 2 < 3$ in $\{-1, -2, -2, 1, -5, 0\}$
11. $3x + 4 < -4$ in $\{-5, -4, -3, -2, -1, 0\}$
12. $-x - 4 < 2$ in $\{26, -29, 1, -19, -26, -5\}$

3.10 Multi-Step Linear Inequalities

Remember that adding and subtracting with inequalities follow the same rules as equations. When you multiply or divide both sides of an inequality by the same positive number, the rules are the same as for equations. However, when you multiply or divide both sides of an inequality by a **negative** number, you must **reverse** the inequality symbol.

Example 16: $-x > 4$

$(-1)(-x) < (-1)(4)$

$x < -4$

Example 17: $-4x < 2$

$\dfrac{-4x}{-4} > \dfrac{2}{-4}$

$x > -\dfrac{1}{2}$

Reverse the symbol when you multiply or divide by a negative number.

When solving multi-step inequalities, first add and subtract to isolate the term with the variable. Then multiply and divide.

Example 18: $2x - 8 > 4x + 1$

Step 1: Add 8 to both sides.

$2x - 8 + 8 > 4x + 1 + 8$

$2x > 4x + 9$

Step 2: Subtract $4x$ from both sides.

$2x - 4x > 4x + 9 - 4x$

$-2x > 9$

Step 3: Divide by -2. Remember to change the direction of the inequality sign.

$\dfrac{-2x}{-2} < \dfrac{9}{-2}$

$x < -\dfrac{9}{2}$

5.13 Multiplying Two Binomials

When you multiply two binomials such as $(x+6)(x-7)$, you must multiply each term in the first binomial by each term in the second binomial. The easiest way is to use the **FOIL** method. If you can remember the word **FOIL**, it can help you keep order when you multiply. The "**F**" stands for **first**, "**O**" stands for **outside**, "**I**" stands for **inside**, and "**L**" stands for **last**.

F		**O**		**I**		**L**
FIRST		**OUTSIDE**		**INSIDE**		**LAST**
Multiply the **first** terms in each binomial		Next, multiply the **outside** terms.		Then, multiply the **inside** terms.		Last, multiply the **last** terms.
$(x+6)(x-7)$		$(x+6)(x-7)$		$(x+6)(x-7)$		$(x+6)(x-7)$
$x \times x = x^2$		$x \times -7 = -7x$		$6 \times x = 6x$		$6 \times -7 = -42$
x^2	$+$	$-7x$	$+$	$6x$	$+$	-42

Now just combine like terms, $6x - 7x = -x$, and write your answer.
$(x+6)(x-7) = x^2 - x - 42$.

Note: It is customary for mathematicians to write polynomials in descending order. That means that the term with the highest-number exponent comes first in a polynomial. The next highest exponent is second and so on. When you use the **FOIL** method, the terms will always be in the customary order. You just need to combine like terms and write your answer.

Multiply the following binomials.

1. $(y-8)(y+3)$
2. $(4x+5)(x+20)$
3. $(5b-3)(3b-5)$
4. $(6g+4)(g-20)$
5. $(8k-7)(-5k-3)$
6. $(9v-4)(3v+5)$
7. $(20p+4)(5p+3)$
8. $(3h-20)(-4h-7)$
9. $(w-5)(w-8)$
10. $(6x+2)(x-4)$
11. $(7t+3)(4t-2)$
12. $(5y-20)(5y+20)$
13. $(a+6)(3a+7)$
14. $(3z-9)(z-5)$
15. $(7c+4)(6c+7)$

6.7 Factoring the Difference of Two Squares

Let's give an example of a **perfect square**.

25 is a perfect square because $5 \times 5 = 25$
49 is a perfect square because $7 \times 7 = 49$

Any variable with an even exponent is a perfect square.

y^2 is a perfect square because $y \times y = y^2$
y^4 is a perfect square because $y^2 \times y^2 = y^4$

When two terms that are both perfect squares are subtracted, factoring those terms is very easy. To factor the difference of perfect squares, you use the square root of each term, a plus sign in the first factor, and a minus sign in the second factor.

Example 11: Factor $4x^2 - 9$

This example has two terms which are both perfect squares, and the terms are subtracted.

Step 1: $(2x \quad 3)\,(2x \quad 3)$

Find the square root of each term. Use the square roots in each of the factors.

Step 2: $(2x + 3)\,(2x - 3)$

Use a plus sign in one factor and a minus sign in the other factor.

Check: Multiply to check. $(2x + 3)\,(2x - 3) = 4x^2 - 6x + 6x - 9 = 4x^2 - 9$

The inner and outer terms add to zero.

Example 12: Factor $81y^4 - 1$

Step 1: $(9y^2 + 1)\,(9y^2 - 1)$

Factor like the example above. Notice, the second factor is also the difference of two perfect squares.

Step 2: $(9y^2 + 1)\,(3y + 1)\,(3y - 1)$

Factor the second term further. **Note: You cannot factor the sum of two perfect squares.**

Check: Multiply in reverse to check your answer.
$(9y^2 + 1)\,(3y + 1)\,(3y - 1) = (9y^2 + 1)\,(9y^2 - 3y + 3y - 1) =$
$(9y^2 + 1)\,(9y^2 - 1) = 81y^4 + 9y^2 - 9y^2 - 1 = 81y^4 - 1$

7.5 Real-World Quadratic Equations

The most common real life situation that would use a quadratic equation is the motion of an object under the force of gravity. Two examples are a ball being kicked into the air or a rocket being shot into the air.

Example 7: A high school football player is practicing his field goal kicks. The equation below represents the height of the ball at a specific time.

$s = -9t^2 + 45t$

$t =$ amount of time in seconds

$s =$ height in feet

Question 1: Where will the ball be at 4 seconds?

Solution 1: Since there are only two variables, you will only need the value of one variable to solve the problem. Simply plug in the number 4 in place of the variable t and solve the equation as shown below.

$s = -9(4)^2 + 45(4)$

$s = -9(16) + 180$

$s = -144 + 180$

$s = 36$

At 4 seconds the ball will be 36 ft in the air.

Question 2: If the ball is 54 ft in the air, how much time has gone by?

Solution 2: This question is similar to the previous one, except that the given variable is different. This time you would be replacing s with 54 and then solve the equation.

$54 = -9t^2 + 45t$ Subtract 54 on both sides.

$0 = -9t^2 + 45t - 54$ Divide the entire equation by -9.

$0 = t^2 - 5t + 6$ Factor the equation.

$0 = (t - 3)(t - 2)$ Solve for t.

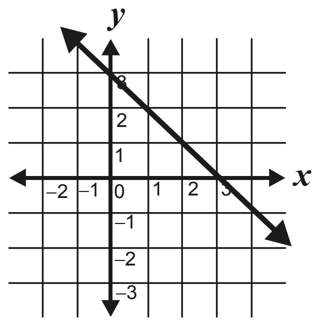
$t = 3 \qquad t = 2$

For this question we got 2 answers. The ball is 54 ft in the air when 2 and 3 seconds have gone by.

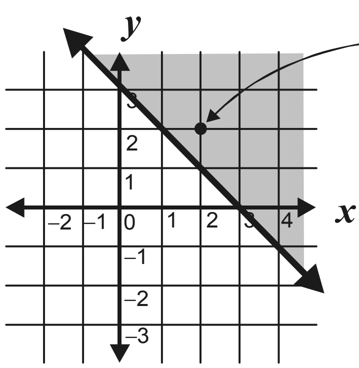
Example 14: Graph $x + y \geq 3$.

Step 1: First, we graph $x + y \geq 3$ by changing the inequality to an equality. Think of ordered pairs that will satisfy the equation $x + y = 3$. Then, plot the points, and draw the line. As shown below, this line divides the Cartesian plane into 2 half-planes, $x + y \geq 3$ and $x + y \leq 3$. One half-plane is above the line, and the other is below the line.

x	y
2	1
0	3
3	0
4	-1



Step 2: To determine which side of the line to shade, first choose a test point. If the point you choose makes the inequality true, then the point is on the side you shade. If the point you choose does not make the inequality true, then shade the side that does not contain the test point.



For our test point, let's choose $(2, 2)$. Substitute $(2, 2)$ into the inequality.

$x + y \geq 3$
$2 + 2 \geq 3$

$4 \geq 3$ is true, so shade the side that includes this point.

Use a solid line because of the $\geq$ sign.

Graph the following inequalities on your own graph paper.

1. $x + y \leq 4$
2. $x + y \geq 3$
3. $x \geq 5 - y$
4. $x \leq 1 + y$
5. $x - y \geq -2$
6. $x < y + 4$
7. $x + y < -1$
8. $x - y \leq 0$
9. $x \geq y + 2$
10. $x < -y + 1$
11. $-x + y > 1$
12. $-x - y < -2$

10.6 Solving Word Problems with Pairs of Equations

Certain word problems can be solved using systems of equations.

Example 7: In a game show, Andre earns 6 points for every right answer and loses 12 points for every wrong answer. He has answered correctly 12 times as many as he has missed. His final score was 120. How many times did he answer correctly?

Step 1: Let r = number of right answers. Let w = number of wrong answers.

We know 2 sets of information that can be made into equations with 2 variables.

He earns $+6$ points for right answers and loses 12 points for wrong answers.

His wins and losses = 120

$$\begin{aligned} 6r - 12w &= 120 \\ 12w &= r \end{aligned}$$

12 times the number of wrong answers = the number of right answers.

Step 2: Substitute the value for r $(12w)$ in the first equation.

$$\begin{aligned} 6(12w) - 12w &= 120 \\ w &= 2 \end{aligned}$$

Step 3: Substitute the value for w back in the equation.

$$\begin{aligned} 6r - 12(2) &= 120 \\ r &= 24 \end{aligned}$$

Example 8: Ms. Sudberry bought pencils and stickers for her first grade class on two different days. The pencils and stickers cost the same each time she went to the store. How much did she pay for each pencil?

	Pencils	Stickers	Total Cost
Tuesday	30	40	\$47.50
Saturday	60	5	\$20.00

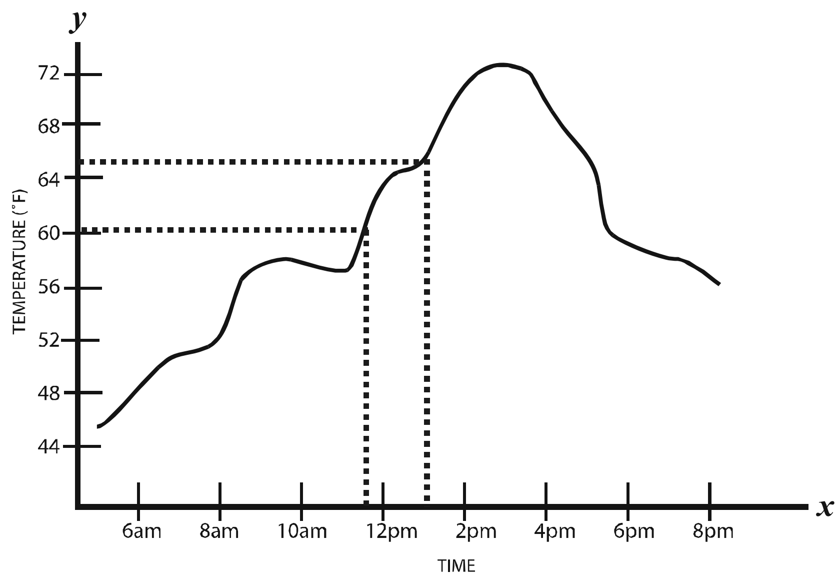
Step 1: Set up your two equations. Let the price of pencils equal x, and the price of stickers equal y.
The amount of the pencils times the price of pencils (x) plus the amount of the stickers times the price of stickers (y) equals the total amount paid for both pencils and stickers.
Equation 1: $30x + 40y = \$47.50$
Equation 2: $60x + 5y = \$20.00$

Step 2: Solve the equations by using one of the methods taught in this chapter. We will use the adding and subtracting method. First, multiply equation 1 by -2, so x will have the same coefficient in each equation but with opposite signs.
$-2(30x + 40y = \$47.50) = -60x - 80y = -\$95.00$

11.5 Interpreting a Graph Representing a Given Situation

Real-world situations are sometimes modeled by graphs. Although an equation cannot be written for most of these graphs, interpreting these graphs provides valuable information. Situations may be represented on a graph as a function of time, length, temperature, etc.

The graph below depicts the temperature of a pond at different times of the day. Refer to the graph as you read through examples 1 and 2.



Example 8: If it is known that a specific breed of fish is most active in waters between 60°F and 65°F, what time of the day would this fish be the most active in this particular pond?

To find the answer, draw lines from the 60°F and 65°F points on the y-axis to the graph. Then, draw vertical lines from the graph to the x-axis. The time range between the two vertical lines on the x-axis indicates the time that the fish are most active. It can be determined from the graph that the fish are most active between 11:30 am and 1:00 PM.

Example 9: Describe the way the temperature of the pond acts as a function of time.

At 6:00 AM, the temperature of the pond is about 47°F. The temperature increases relatively steadily throughout the morning and early afternoon. The temperature peaks at 72°F, which is around 2:30 PM during the day. Afterwards, the temperature of the pond starts to decrease. The later it gets in the evening, the more the temperature of the water decreases. The graph shows that at 8 PM the temperature of the pond is about 57°F.

12.3 Logic

A **conditional statement** is a type of logical statement that has two parts, a **hypothesis** and a **conclusion**. The statement is written in "if-then" form, where the "if" part contains the hypothesis and the "then" part contains the conclusion. For example, let's start with the statement "Two lines intersect at exactly one point." We can rewrite this as a conditional statement in "if-then" form as follows:

$$\underbrace{\text{If two lines intersect}}_{\text{hypothesis}}\text{, then }\underbrace{\text{their intersection is at exactly one point}}_{\text{conclusion}}.$$

Conditional statements may be true or false. To show that a statement is false, you need only to provide a single **counterexample** which shows that the statement is not always true. To show that a statement is true, on the other hand, you must show that the conclusion is true for all occasions in which the hypothesis occurs. This is often much more difficult.

Example 3: Provide a counterexample to show that the following conditional statement is false:
If $x^2 = 4$, then $x = 2$.

To begin with, let $x = -2$.
The hypothesis is true, because $(-2)^2 = 4$.
For $x = -2$, however, the conclusion is false even though the hypothesis is true.
Therefore, we have provided a counterexample to show that the conditional statement is false.

The **converse** of a conditional statement is an "if-then" statement written by switching the hypothesis and the conclusion. For example, for the conditional statement "If a figure is a quadrilateral, then it is a rectangle," the converse is "If a figure is a rectangle, then it is a quadrilateral."

The **inverse** of a conditional statement is written by negating the hypothesis and conclusion of the original "if-then" conditional statement. Negating means to change the meaning so it is the negative, or opposite, of its original meaning. The inverse of the conditional statement "If a figure is a quadrilateral, then it is a rectangle" is "If a figure is **not** a quadrilateral, then it is **not** a rectangle."

The **contrapositive** of a conditional statement is written by negating the converse. That is, switch the hypothesis and conclusion of the original statement, and make them both negative. The contrapositive of the conditional statement "If a figure is a quadrilateral, then it is a rectangle" is "If a figure is not a rectangle, then it is not a quadrilateral."

Practice Test 1

Session 1

1. Simplify $\dfrac{x+7}{7x^2-343}$.

A $\dfrac{1}{2}$

B $\dfrac{1}{7\left(x^2-343\right)}$

C $\dfrac{1}{7\left(x+7\right)}$

D $\dfrac{1}{7\left(x-7\right)}$

A1.7.1

2. Solve the proportion $\dfrac{3x+2}{2}=\dfrac{3x-12}{6}$.

A $x=1$
B $x=-1$
C $x=-3$
D $x=-6$

A1.7.2

3. Use the substitution method to solve the pair of equations $y=3x+11$ and $y=2x$.

A $(-1,-2)$
B $(1,2)$
C $(-11,-22)$
D $(11,22)$

A1.5.3

4. Which property is demonstrated below?

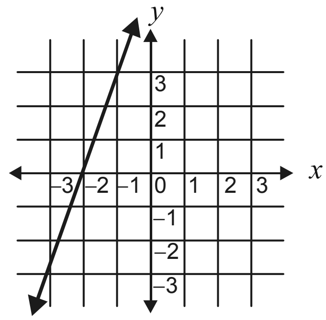
$(a\times b)\times c=a\times(b\times c)$

A Associative Property of Multiplication
B Commutative Property of Multiplication
C Identity Property of Multiplication
D Inverse Property of Multiplication

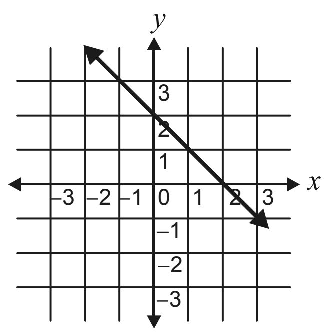
A1.1.3

5. Which is the graph of $x-3y=6$?

A



B



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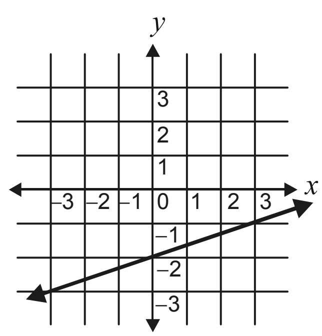

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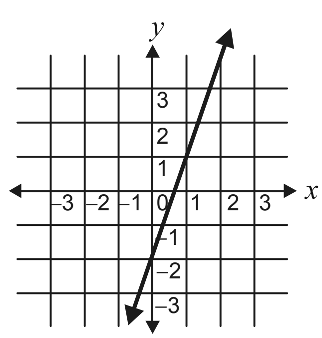


A1.4.1

Practice Test 2

Session 1

1. Use addition to solve the equations: $x - y = 1$ and $x + y = 6$.

   A $\left(\frac{7}{4}, \frac{3}{4}\right)$

   B $\left(\frac{10}{3}, \frac{8}{3}\right)$

   C $\left(\frac{5}{2}, \frac{7}{2}\right)$

   D $\left(\frac{7}{2}, \frac{5}{2}\right)$

   A1.5.4

2. Solve the algebraic proportion $\dfrac{x+2}{7} = \dfrac{x+4}{21}$.

   A $x = -1$
   B $x = 2$
   C $x = -4$
   D $x = 7$

   A1.7.2

3. What is the domain of $y = x^3 - 7$?

   A $-7 \leq x < \infty$
   B $-\infty < x < \infty$
   C $-\infty < y \leq -7$
   D $-7 \leq y < \infty$

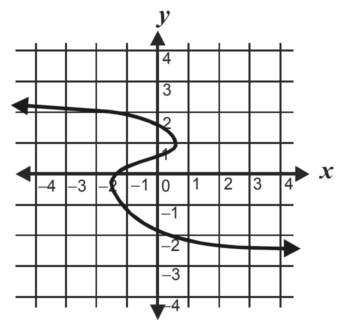
   A1.3.4

4. What is the range of the function $y = -x^2 + 1$?

   A $-\infty < y < \infty$
   B $-\infty < y \leq 0$
   C $-\infty < y \leq 1$
   D $1 \leq y < \infty$

   A1.3.4

5. Is the graph below a function?



   A Yes, because it passes the vertical line test.
   B Yes, because it passes the horizontal line test.
   C No, because it fails the vertical line test.
   D No, because it fails the horizontal line test.

   A1.3.3

6. What is 790 centimeters per second in kilometers per hour?

   A 28.44 km/hr
   B $28,440$ km/hr
   C 0.2844 km/hr
   D 0.0079

   A1.1.5

7. What is the slope of a line represented by the equation $x = 2y - 3$?

   A -2

   B $-\frac{1}{2}$

   C $\frac{1}{2}$

   D 2

   A1.4.2